MODELING PROBLEMS ALGEBRAICALLY

3

Name: Key

SECTION 3.5 EQUATIONS AND INEQUALITIES ON NUMBER LINE

VOCABULARY

DEFINITION	EXAMPLE
Inequality: a statement that one expression is always less than (or greater than) another	$3 < x + 2$ $y > a$

**Big Idea:** How do we solve equations and inequalities on a number line?

EXPLORATION 1

Suppose a and x are numbers located on the number line as seen below. Locate and label the points that represent the indicated numbers. Use string to act out how you determine your answer.

1.   Plot points that represents each of the following: 2a, 3a, -a, -2a, -3a
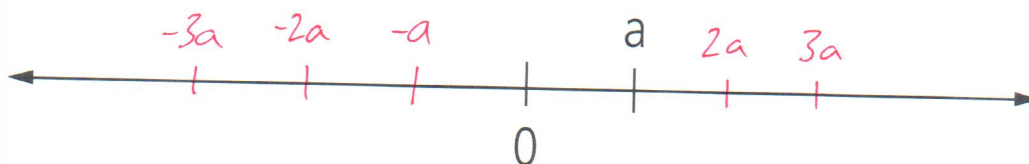
2.   Plot points that represents each of the following: 2x, 3x, -x, -2x, -3x
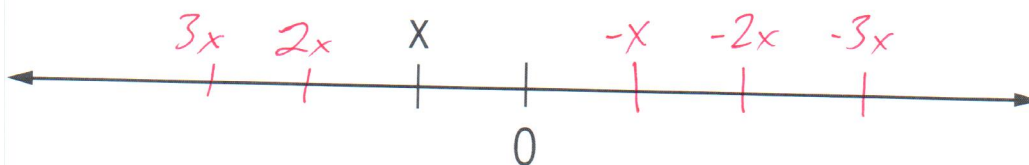
3. Compare the results from parts 1 and 2. What do you notice?

"-a"s are on the left, but "-x"s are on the right of 0.

EXPLORATION 2

PART A: Suppose X is a number that is located on the number line as seen below. Locate and label the points that represent the indicated expressions. The numbers 0 and 1 are also labeled. Plot a point that represents each of the following expressions:

$$x + 1, x + 2, x - 1, x - 2, \frac{x}{2}, 1 - x$$
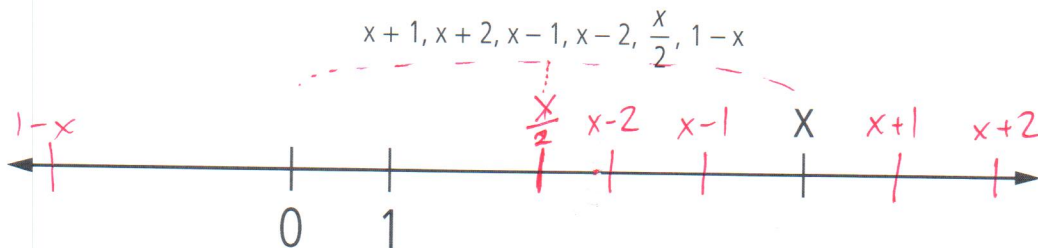
PART B: Suppose we know the location of each of the expressions as indicated on the number line below. Find the locations for a, b and c. Explain how you locate each of these points on the number line.
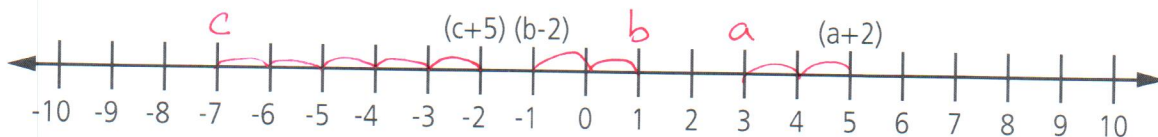

In Part A in this exploration we used the location of a variable on the number line to locate expressions on the same number line. In Part B, we were given the location of an expression, such as $a + 2 = 5$, and used it to find the location of the variable a on the number line. We see that $a = 3$. In other words, we solved the equation using the number line.
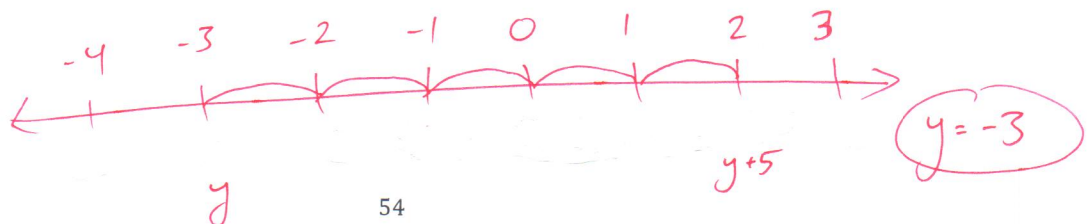
PROBLEM 1

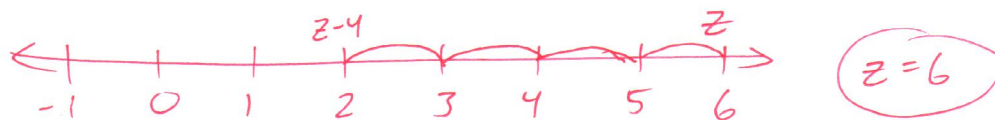1. Use the number line to solve each of the following equations:

a. $x + 3 = 5$

$x = 2$

b. $y + 5 = 2$

$y = -3$

c. $z - 4 = 2$

$z = 6$
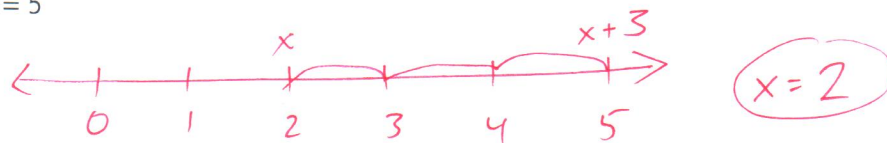
d. Discuss how solving these equations on the number line compares with the balance scale method.

answers may vary: On the number line we move left or right, while on the balance scale we add or remove weights.

EXAMPLE 1: Translate the following into mathematical expressions

1. The number of apples, x, consumed is more than twice the number of bananas, y.

2. Bob's age, B, is less than 35 years.

3. The cost of three apples is less than $2.00. Write an inequality to represent the possible cost A of one apple.
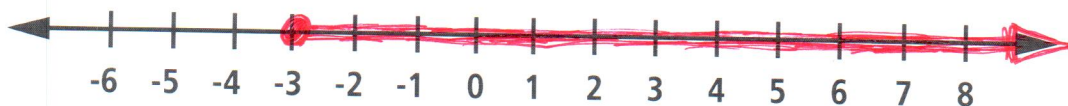
1. $x > 2y$

2. $B < 35$

3. $3A < 2$ so $A < \frac{2}{3}$

EXAMPLE 2

On the number line below represent the set T of all numbers x such that $-3 \leq x$.

This is the same as $x \geq -3$.

How do you indicate that -3 is in the set?

Shade a circle around that point.

EXAMPLE 3

Draw a graphical solution to the inequality $x - 4 < 8$ on the number line below.

$x - 4 + 4 < 8 + 4$ so $x < 12$.

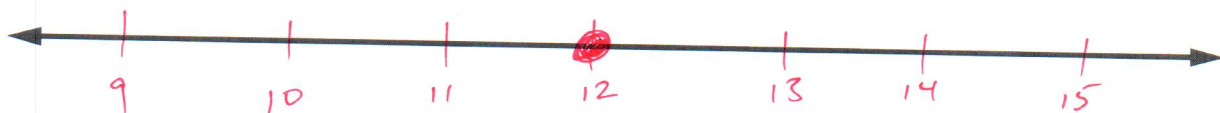
PRACTICE EXERCISES:

1. Draw a graphical solution for the following equations:
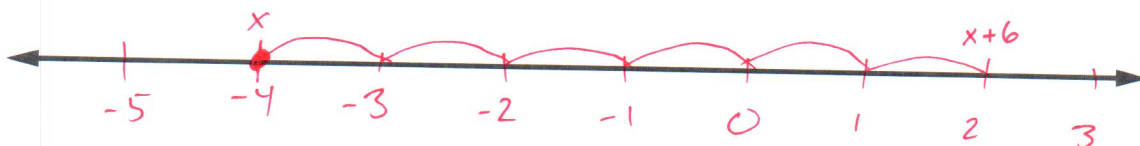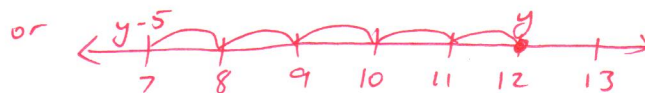
   a. $y - 5 = 7$ $y - 5 + 5 = 7 + 5$ $y = 12$

   or

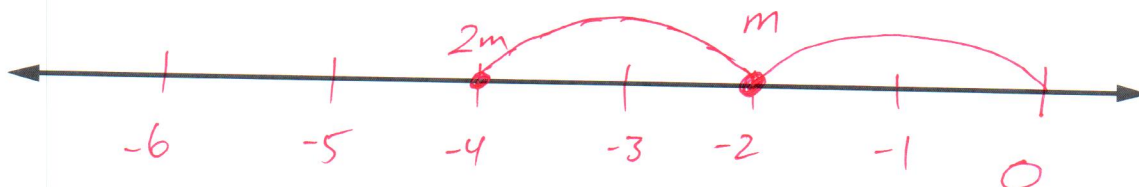   b. $x + 6 = 2$ $x + 6 - 6 = 2 - 6$
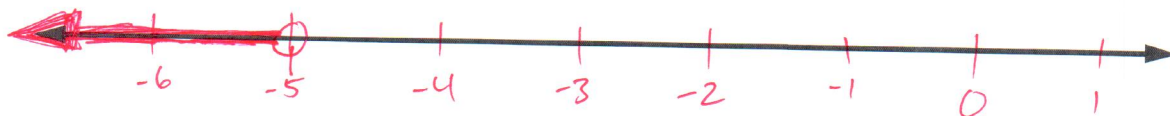 $x = -4$

   c. $2m = -4$ $\dfrac{(2m)}{2} = \dfrac{-4}{2}$ $m = -2$

2. Draw a graphical solution for the following inequalities:
   a. $w + 3 < -2$ $w + 3 - 3 < -2 - 3$ $w < -5$

   b. $x - 4 \leq 3$ $x - 4 + 4 \leq 3 + 4$ $x \leq 7$

3. Graph the solution sets for the following inequalities:

   a. $3 < x < 7$

   b. $-2 < x + 1 < 3$

   $\begin{array}{c} -2 < x + 1 \\ -1 \quad\quad -1 \\ -3 < x \end{array}$ $\begin{array}{c} x + 1 < 3 \\ -1 \quad -1 \\ x < 2 \end{array}$

SUMMARY (What I learned today)