VARIABLES, EXPRESSIONS AND EQUATIONS

1

SECTION 1.1   CONSTRUCTING A NUMBER LINE

Name: _____Key_____   Date: _____   Period: _____

Vocabulary

| DEFINITION	EXAMPLE
Integers all of the negative integers, zero, and the positive integers	..., $-4, -3, -2, -1, 0, 1,$ $2, 3, 4, ...$
Natural Numbers (also known as counting numbers or positive integers) $1, 2, 3, ...$	$3, \quad 7, \quad 18$ $1, 2, 3, ...$
Whole Numbers natural numbers with zero	$0, 1, 2, 3, ...$
Rational Numbers a a number $\frac{a}{b}$ where a is an integer and b is a natural number	$\dfrac{-3}{4}, \quad \dfrac{6}{5}$
Absolute Value a number's distance from zero. $\|x\| = \begin{cases} -x, \text{ if } x < 0 \\ x, \text{ if } x \geq 0 \end{cases}$	$\|-3\| = 3$ $\|3\| = 3$

EXPLORATION 1

Build a number line.

1. Make a number line on a large piece of paper. Put the number 1 in the middle of the line. Locate and label the first twenty natural numbers.

2. Plot and label 0 on the number line. If we include 0 and the natural numbers, what do we call the set of numbers?

3. Using a red marker, plot and label the negative integers from −1 to −20.

4. Using a different colored marker, plot and label 3 fractions between each of the following pairs of integers:

2 and 3	4 and 5	− 1 and 0	− 3 and − 2

answers will vary: $\frac{9}{4}, \frac{12}{5}, \frac{8}{3}$ $\frac{17}{4}, \frac{19}{4}, \frac{9}{2}$ $\frac{-1}{2}, \frac{-3}{4}, \frac{-18}{23}$ $\frac{-9}{4}, \frac{-12}{5}, \frac{-7}{3}$

EXAMPLE 1

Create a Venn Diagram to show the relationship between the following sets of numbers:

*   rational numbers
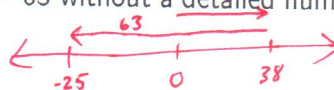*   whole numbers
*   integers
*   natural numbers
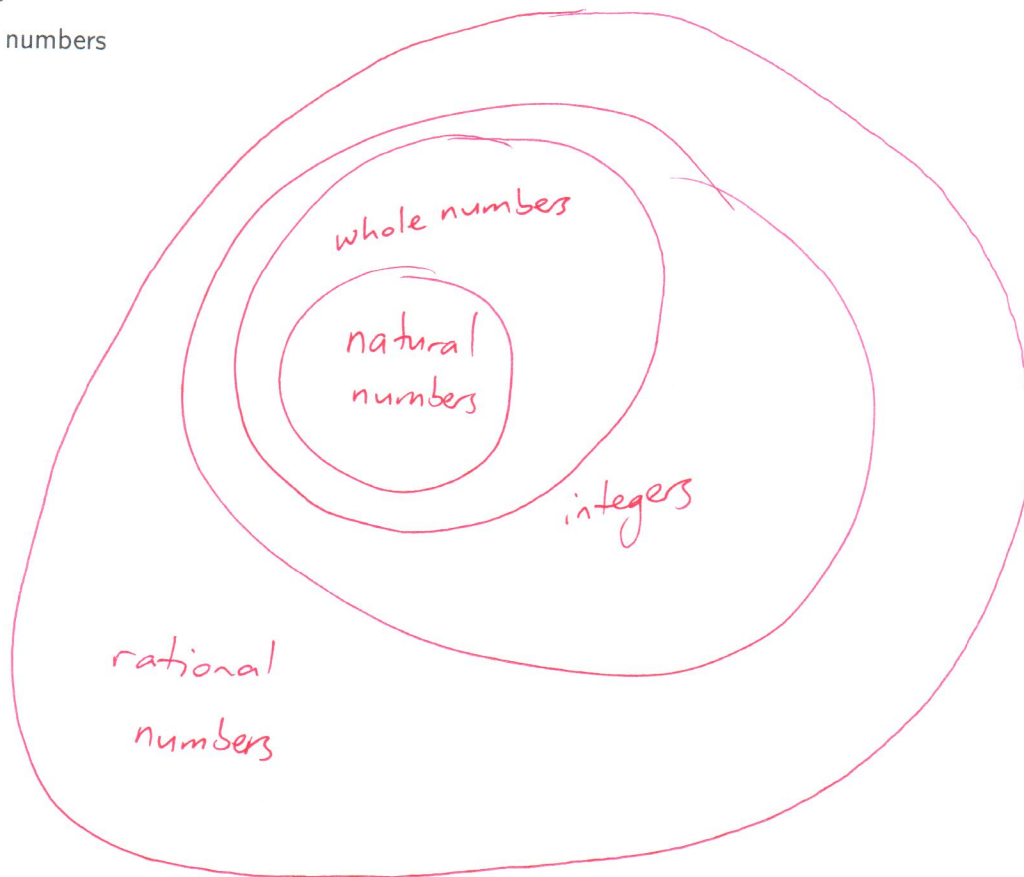
EXPLORATION 4

1. Use the number line to illustrate the sum $3 + (-4)$ and the difference $3 - 4$. Explain how you arrived at your answer and location for each problem. Then, using the same pattern, explain how you compute the sum $38 + (-63)$ and the difference $38 - 63$ without a detailed number line.
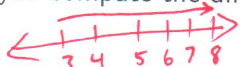
   $3 + (-4) = -1$
   $3 - 4 = -1$

   $38 - 63 = 25$
   $38 + (-63) = -25$

   Move 38u to 0, then 25 more. (since
   $38 + 25$
   $= 63$)

2. Use the number line to illustrate the difference $3 - (-5)$ and sum $3 + 5$. Then explain how you compute the difference $38 - (-63)$ without a detailed number line.

   $3 - (-5) = 3 + 5 = 8$

   move left backwards = moving right

   $38 - (-63) = 38 + 63 = 101$

3. Summarize the rules for addition and subtraction of integers.

   Adding: go "forward" on a number line.
   Subtraction: go "backward" on a number line.

4. Use the number line to illustrate the product $3(-4)$ and $-3(4)$. Explain how you arrived at your answer and location for each problem. Then using the same pattern, explain how you compute the products $18(-6)$ and $-5(12)$ without a detailed number line.
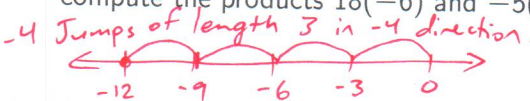
   -4 Jumps of length 3 in -4 direction. 4 Jumps of length -3.

   -6 jumps of 18 = -108
   12 jumps of -5 = -60

5. Use the number line to illustrate the product $-3(-4)$. Explain how you arrived at your answer and location for each problem. Then using the same pattern, explain how you compute the product $-28(-3)$.

   -4 jumps of length -3 = 12

   -3 jumps of length -28 = 84

6. Summarize the rules for multiplication of integers.

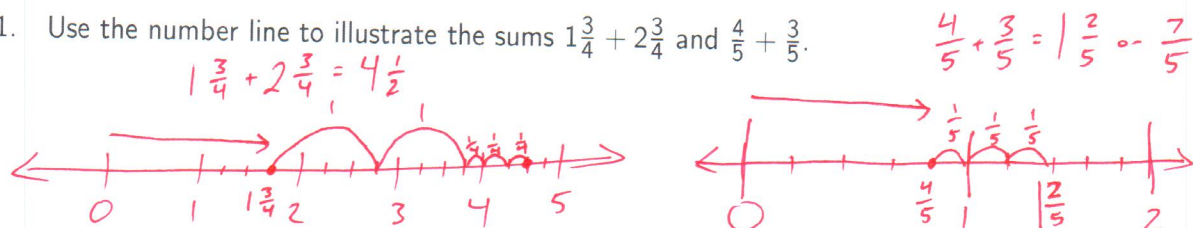   positive × negative = negative
   negative × positive = negative

   positive × positive = positive
   negative × negative = positive

EXPLORATION 5

1. Use the number line to illustrate the sums $1\frac{3}{4} + 2\frac{3}{4}$ and $\frac{4}{5} + \frac{3}{5}$.

$$\frac{4}{5} + \frac{3}{5} = 1\frac{2}{5} \text{ or } \frac{7}{5}$$

$$1\frac{3}{4} + 2\frac{3}{4} = 4\frac{1}{2}$$

2. Starting at the point representing 3, determine and locate on the number line the following numbers. Explain how you arrived at your answer.

   a. The number that is 5 more than this number.

   add 5

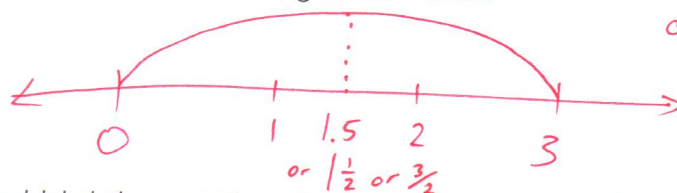   b. The number that is 5 less than this number.

   subtract 5

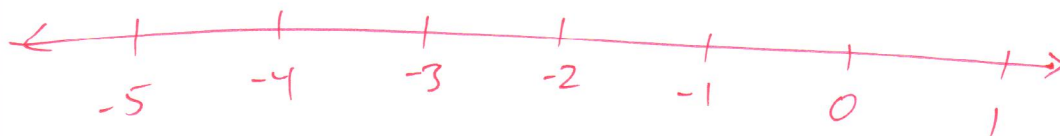   c. The number that is 3 times this number.

   multiply by 3

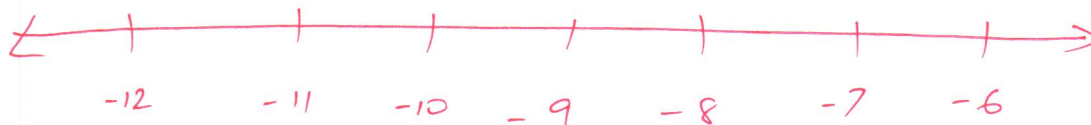   d. The number that is half as big as this number.

   divide by 2

3. Locate and label three numbers that are greater than −5. Locate and label three numbers that are less than −6.

(answers will vary, and may be fractional.)

EXPLORATION 6

Use your number line to determine the distance between 6 and 13. How did you arrive at your answer.

7, by counting from 6 to 13

1. What is the distance from 12 to 4? Explain how you got your answer.

   8, count down from 12 to 4 or subtract

2. What is the distance from -3 to -11? From -9 to -2? Explain how you got your answers.

   8   7

   counting on the number line or subtracting

3. What is the distance from -7 to 4? What is the distance from 5 to -7? Explain how you got your answers.

   counting, or adding without signs (i.e., 7+4)
   11   12
   (students may use absolute value)

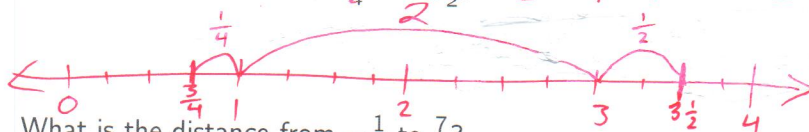4. Find the distance between $\frac{1}{2}$ and $3\frac{1}{2}$.

   3

5. Find the distance between $\frac{1}{2}$ and $\frac{3}{4}$.
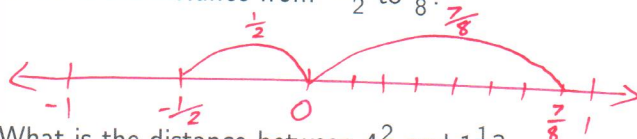
   $\frac{1}{4}$

6. Find the distance between $\frac{3}{4}$ and $3\frac{1}{2}$.

   $3\frac{1}{2} - \frac{3}{4} = 2 + 1 + \frac{1}{2} - \frac{3}{4} = 2 + \frac{3}{4} + \frac{1}{4} + \frac{1}{2} - \frac{3}{4}$

   $= 2 + \frac{1}{4} + \frac{1}{2} = \boxed{2\frac{3}{4}}$

   $2 + \frac{1}{4} + \frac{1}{2} = 2\frac{3}{4}$

7. What is the distance from $-\frac{1}{2}$ to $\frac{7}{8}$?

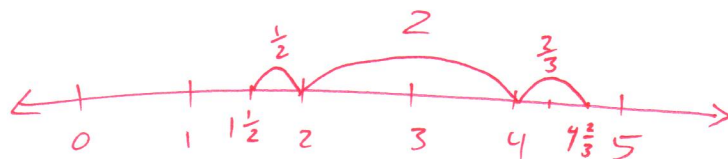   *$\frac{1}{2} + \frac{7}{8} = \frac{4}{8} + \frac{7}{8} = \frac{11}{8}$*

8. What is the distance between $4\frac{2}{3}$ and $1\frac{1}{2}$?

   $4\frac{2}{3} - 1\frac{1}{2} = 4 + \frac{2}{3} - \left(1 + \frac{1}{2}\right) = 4 + \frac{2}{3} - 1 - \frac{1}{2} = 4 - 1 + \frac{2}{3} - \frac{1}{2}$

   $= 3 + \frac{4}{6} - \frac{3}{6} = 3 + \frac{1}{6} = 3\frac{1}{6}$

   OR

   $2 + \frac{1}{2} + \frac{2}{3} = 2 + \frac{3}{6} + \frac{4}{6}$

   $= 2 + \frac{7}{6} = 2 + 1 + \frac{1}{6}$

   $= \boxed{3\frac{1}{6}}$

SUMMARY (What I learned today)