ME1 Section 7.4: Proportions

Vocab: Proportion: an equation of ratios (of the form $\frac{a}{b} = \frac{c}{d}$, where b and d are not equal to zero)

Example: $\frac{x \text{ miles}}{2 \text{ inches}} = \frac{50 \text{ miles}}{1 \text{ inch}}$

Exploration 1: ... does 2 inches represent? 100 miles

Example 1:

time in minutes	# of leaves cut
0	0
7	4
14	8
21	12
28	16
35	20

... 35 min corresponds to 20 leaves cut...

Unit Rate: 20 leaves

Proportion: variable is "L"

... multiply by 35 then divide by 35? (answers may vary slightly)

nothing, they cancel, the L is by itself

$$35 \text{ min} \cdot \frac{L \text{ leaves}}{35 \text{ minutes}} = \frac{4 \text{ leaves}}{7 \text{ minutes}} \cdot 35 \text{ min}$$

$$L \text{ leaves} = \frac{4}{7} \cdot 35 \text{ leaves}$$

$$L \text{ leaves} = 20 \text{ leaves}$$

$$L = 20$$

Example 2: $\frac{\$2.79}{3 \text{ bags}} = \frac{C \text{ dollars}}{7 \text{ bags}}$ (① ②)

$7 \text{ bags} \cdot \frac{2.79 \text{ dollars}}{3 \text{ bags}} = \frac{C \text{ dollars}}{7 \text{ bags}} \cdot 7 \text{ bags}$

$7(.93) \text{ dollars} = C \text{ dollars}$

$\boxed{C = \$6.51}$

unit rate: $\frac{\$2.79}{3 \text{ bags}} = \$0.93 \text{ per bag}$

$.93 \frac{\text{dollars}}{\text{bag}} \cdot 7 \text{ bags} = 7 \cdot 0.93 \text{ dollars} = \boxed{\$6.51}$

Problems:

$\frac{2}{7}$ $\frac{30}{126} = \frac{5}{21}$ $\frac{2}{7} \neq \frac{5}{21}$	$\frac{24}{8} = \frac{3}{1}$ $\frac{6}{2} = \frac{3}{1}$ $\frac{24}{8} = \frac{3}{1} = \frac{6}{2}$	$\frac{6}{20} = \frac{3}{10}$ $\frac{3}{10}$ $\frac{6}{20} = \frac{3}{10}$
$\frac{9}{6} = \frac{3}{2}$ $\frac{6}{2} = \frac{3}{1}$ $\frac{3}{2} \neq \frac{3}{1}$	$\frac{3}{27} = \frac{1}{9}$ $\frac{5}{125} = \frac{1}{25}$ $\frac{1}{9} \neq \frac{1}{25}$	$\frac{5}{11}$ $\frac{11}{5}$ $\frac{5}{11} \neq \frac{11}{5}$
$\frac{5}{27}$ $\frac{30}{162} = \frac{5}{26}$ $\frac{5}{27} = \frac{30}{162}$	$\frac{4}{7}$ $\frac{60}{105} = \frac{4}{7}$ $\frac{4}{7} = \frac{60}{105}$	$\frac{51}{17} = \frac{3}{1}$ $\frac{3}{1}$ $\frac{51}{17} = \frac{3}{1}$

Example 3: (processes may vary)

$\boxed{\frac{6}{14} = \frac{x}{20}}$ $20 \cdot \frac{6}{14} = \frac{x}{20} \cdot 20$ $\boxed{x = \frac{60}{7}} \approx 8.6$	$43 \cdot \boxed{\frac{1}{9} = \frac{x}{43}} \cdot 43$ $x = \frac{43}{9} \approx 4.8$	$11 \cdot \boxed{\frac{x}{11} = \frac{7}{17}} \cdot 11$ $x = \frac{77}{17} \approx 4.5$
$16 \cdot \boxed{\frac{x}{16} = \frac{14}{5}} \cdot 16$ $x = \frac{14 \cdot 16}{5} = \frac{224}{5}$ ≈ 44.8	$2.2 \cdot \boxed{\frac{1.5}{8} = \frac{x}{2.2}} \cdot 2.2$ $\frac{3.3}{8} = x = \frac{33}{80} \approx 4.1$	$21 \cdot \boxed{\frac{x}{21} = \frac{16}{31}} \cdot 21$ $x = \frac{21 \cdot 16}{31} = \frac{336}{31}$ ≈ 10.8

Exploration 3: Step 1: $\frac{1 \text{ inch}}{50 \text{ miles}}$ (answers for this exploration will vary)

Step 2: 3 inches

Step 3: $\frac{x \text{ miles}}{3 \text{ inches}} = \frac{50 \text{ miles}}{1 \text{ inch}}$ $x = 150$ miles

Step 4: two other cities, repeat steps.

Exploration 4:

country/currency	equivalent to $1	equivalent to $50
USA / dollar	$1	$50
other country/currency (A_{dollar})	.10 A ($\frac{.1}{1} = \frac{x}{50}$)	5 A_{dollar}
another country/currency (B_{dollar})	2.50 B $\frac{2.50}{\$1} = \frac{x}{\$50}$	125 B_{dollar}

these will vary based on exchange rates (made up examples)

100 _____ is worth ____ dollars

Exploration 5: building to shadow = B to 15 = $\frac{B}{15}$

tree to shadow = 12 to 2 = $\frac{12}{2} = \frac{6}{1}$

$15 \cdot \frac{B}{15} = \frac{6}{1} \cdot 15$

$B = \boxed{90 \text{ feet}}$

Problems: 1) $5 \text{ aliens} \cdot \left(\frac{24 \text{ socks}}{3 \text{ aliens}}\right) = \left(\frac{x \text{ socks}}{5 \text{ aliens}}\right) \cdot 5 \text{ aliens}$

$\boxed{40 \text{ socks}} = x \text{ socks}$

2) $3 \text{ red} : 2 \text{ blue} = \frac{3 \text{ red}}{2 \text{ blue}} = \boxed{\frac{3}{2} \frac{\text{red}}{\text{blue}}} = \frac{3}{2}$ red per blue

or $1.5 \frac{\text{red}}{\text{blue}}$

$\left(1.5 \frac{\text{red}}{\text{blue}}\right) 24 \text{ blue} = \boxed{36 \text{ red cars}}$

3) a. 5 blue to 1 white $= \frac{5 \text{ blue}}{1 \text{ white}}$

$11 \text{ white} \cdot \frac{5 \text{ blue}}{1 \text{ white}} = \frac{x \text{ blue}}{11 \text{ white}} \cdot 11 \text{ white}$

$55 \text{ blue} = x \text{ blue}$ $\boxed{55 \text{ blue balloons}}$ in each bouquet

b. $5 \text{ blue} : 2 \text{ silver} = \frac{5 \text{ blue}}{2 \text{ silver}}$

$\frac{5 \text{ blue}}{2 \text{ silver}}$

$2 \text{ silver} : 5 \text{ blue} = \frac{2 \text{ silver}}{5 \text{ blue}}$

$55 \text{ blue} \cdot \frac{2 \text{ silver}}{5 \text{ blue}} = \frac{x \text{ silver}}{55 \text{ blue}} \cdot 55 \text{ blue}$

$22 \text{ silver} = x \text{ silver}$

$\boxed{22 \text{ silver balloons}}$

c. total # of balloons: $\underset{\text{white}}{11} + \underset{\text{blue}}{55} + \underset{\text{silver}}{22} = \boxed{88 \text{ balloons}}$

4) $\frac{22 \text{ miles}}{1 \text{ gallon}}$ OR $\frac{1 \text{ gallon}}{22 \text{ miles}}$

a. $50 \text{ mi.} \cdot \frac{1 \text{ gallon}}{22 \text{ miles}} = \frac{x \text{ gallons}}{50 \text{ miles}} \cdot 50 \text{ miles}$

$\boxed{\frac{25}{11} \text{ gallons}} = x \text{ gallons}$

b. $100 \text{ miles} \cdot \frac{1 \text{ gallon}}{22 \text{ miles}} = \frac{x \text{ gallons}}{100 \text{ miles}} \cdot 100 \text{ miles}$

(Note: double miles means double gas)

$\boxed{\frac{50}{11} \text{ gallons}} = x \text{ gallons}$

c. $2 \text{ miles} \cdot \frac{1 \text{ gallon}}{22 \text{ miles}} = \frac{x \text{ gallons}}{2 \text{ miles}} \cdot 2 \text{ miles}$

$\boxed{\frac{1}{11} \text{ gallon}} = x \text{ gallons}$

5) 39 words per minute

$\frac{39 \text{ words}}{1 \text{ minute}}$ so $\frac{1 \text{ minute}}{39 \text{ words}}$

$$500 \text{ words} \cdot \frac{1 \text{ minute}}{39 \text{ words}} = \frac{M \text{ minutes}}{500 \text{ words}} \cdot 500 \text{ words}$$

$$M \text{ minutes} = \frac{500}{39} \text{ minutes} = \boxed{12.8 \text{ minutes}}$$

6) $$6 \text{ tickets} \cdot \frac{\$128}{4 \text{ tickets}} = \frac{\$x}{6 \text{ tickets}} \cdot 6 \text{ tickets}$$

$$\boxed{\$192} = \$x$$

7) $$35 \text{ ft} \cdot \frac{\text{man: } 6 \text{ ft}}{15 \text{ ft shadow}} = \frac{\text{tree: } H \text{ ft}}{35 \text{ ft shadow}} \cdot 35 \text{ ft}$$

$$\boxed{14 \text{ ft}} = 14 \text{ ft}$$