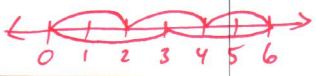
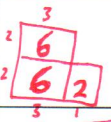
MULTIPLYING AND DIVIDING 2

Name: Key Date: Period:

SECTION 2.2 AREA MODEL FOR MULTIPLICATION

VOCABULARY

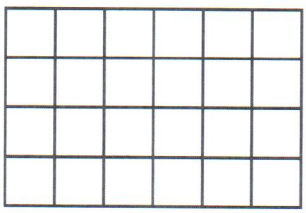
DEFINITION	EXAMPLE
Linear Model: skip counting along a number line to multiply	$2 \cdot 3 = 6 = 3 \cdot 2$ 
Commutative Property of Multiplication: numbers can be multiplied in either order to obtain the same result $x \cdot y = y \cdot x$	$6 \cdot 4 = 24$ $4 \cdot 6 = 24$
Partial Product: part of a whole, found by multiplying	 6, 6, & 2 are partial products (Area = 14)
Distributive Property: $n(k+m) = nk+nm$	$3(2+1) = 3 \cdot 3 = 9$ $3 \cdot 2 + 3 \cdot 1 = 6 + 3 = 9$

Big Idea: How do we use the area model to represent multiplication?

EXPLORATION 1: AREA MODEL FOR MULTIPLICATION

A bird refuge is in the shape of a rectangle 4 miles long and 6 miles wide. Draw a visual representation of this refuge on grid paper, using 1 centimeter = 1 mile, and use it to determine the area of this rectangle. Explain how you use the grid to compute the area. Multiply 4 miles by 6 miles using the traditional algorithm only after you have an answer using the visual representation. count 24 squares

To multiply 4 by 6, consider the picture of the rectangle below. The area of a rectangle is the number of units, or 1×1, squares that it takes to cover the figure with no overlaps and no gaps. What is the area of the rectangle below assuming that each square in the grid has area 1 square unit?

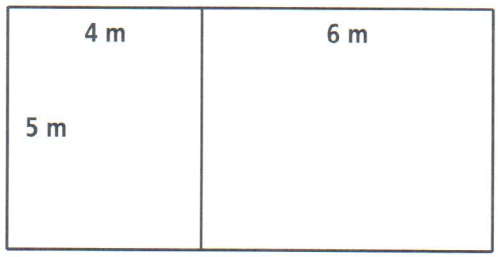


24 square units

The area is 24 square units. The dimensions of this rectangle are 4 units in width and 6 units in length. Sometimes the dimensions are abbreviated and used to describe the rectangle. The rectangle is 4 units wide and 6 units long: there are 4 rows and 6 columns. The area can be computed by summing the areas of the columns: $4 + 4 + 4 + 4 + 4 + 4 = 4 \cdot 6 =$ 24. We can also think of this area as the sum of the area of the rows: $6 + 6 + 6 + 6 = 6 \cdot 4 =$ 24. The rectangle is called a 4 by 6, or a 6 by 4, rectangle because the area is computed as the product $4 \cdot 6 = 6 \cdot 4 =$ 24. We call this relationship the **commutative property of multiplication**.

EXAMPLE 1:

The Elliots are constructing a small building that is one room wide and two rooms long. Each room is five meters wide. The front room is 4 meters long, and the back room is 6 meters long. The floor plan below shows the layout.



1. What is the floor space of each room? little room: $20 m^2$ big room: $30 m^2$

2. What is the floor space of the building? 50 square meters

3. How are the areas of the two rooms related to the area of the building?
the area of the building is the sum of the areas of the rooms

EXAMPLE 2:

Now suppose the dimensions of the Elliots' building have not been decided yet. We need a formula for the areas. Call the width of the building *n* feet, and the lengths of rooms 1 and 2, *k* and *m* feet, respectively. Draw and label the floor plan of the Elliots' building.

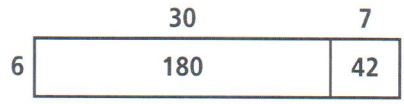
a) What is the floor space of each room? Room 1: nk sq. ft. Room 2: nm sq. ft.

b) What is the floor space of the building? $n(k+m)$ or $nk + nm$ sq. ft.

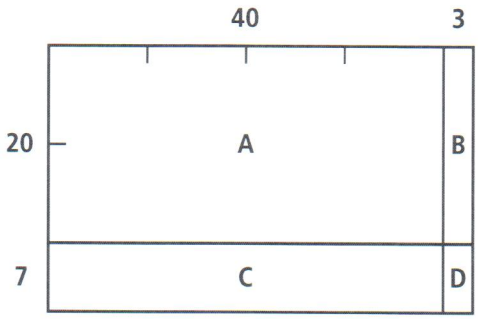
Using Example 2, write a rule for this product: n(k + m) = nk + nm. This rule is called the **distributive property**.

EXPLORATION 2: DISTRIBUTIVE PROPERTY

You have already learned to multiply two-digit and three-digit numbers. Now you can use the area model and the distributive property to explore this process carefully. Begin by modeling the product of a one-digit number and a two-digit number. To multiply 6 · 37, use place value to write the product 6 · 37 as 6 (30+7). By the distributive property, 6 · 37 = 6 (30 + 7) = 6 · 30 + 6 · 7 = 180 + 42 = 222.



Visualize the product of 27 x 43 as area with the picture below:



Area of A = 20 · 40 = 800;
Area of B = 20 · 3 = 60;
Area of C = 7 · 40 = 280;
Area of D = 7 · 3 = 21. The total area is 800 + 60 + 280 + 21 = 1161.

You can extend the same process to multiply 27 by 43 using the distributive property, or in the more traditional vertical format.

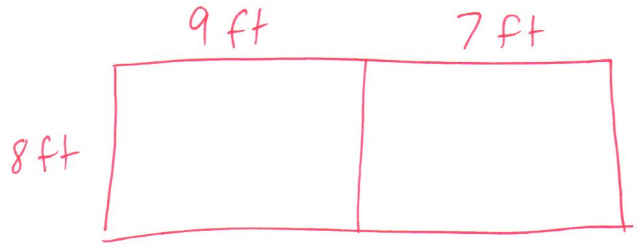
27 · 43 = (20 + 7) (40 + 3)
= 20 (40 + 3) + 7 (40 + 3)
= 20 · 40 + 20 · 3 + 7 · 40 + 7 · 3

= 800 + 60 + 280 + 21 = 1161

$$\begin{array}{r} 27 \\ \times\ 43 \\ \hline 21 \\ 60 \\ 280 \\ +\ 800 \\ \hline \end{array}$$

PROBLEMS:

1. The Luis family is constructing a rectangular building that is one room wide and two rooms long. Each room is 8 feet wide. The front room is 9 feet long, and the back room is 7 feet long. Create the floor plan that shows this situation.



a) What is the floor space of each room? Room 1: $8 \cdot 9 = 72$ sq. ft.
Room 2: $8 \cdot 7 = 56$ sq. ft.

b) What is the floor space of the building? $72 + 56 = 128$ sq ft
OR $8 \cdot (9+7) = 8 \cdot 16 = 128$ sq. ft.

c) How are the areas of the two rooms related to the area of the building?

the sum of the areas of the rooms equals the area of the building

2. Use the area model and the distributive property to compute the following products. Indicate the area of each interior part in your model. The first one is started for you.

a. (47) (29) = 1363

(40+7)(20+9) =

40(20+9) + 7(20+9) =

40·20 +40·9 + 7·20 + 7·9 =

40 7

800 +360 +140 +63 =

1363

20	A 800	B 140
9	360 C	63 D

b. (12) (31) = 372

(10+2)(30+1) =

10(30+1) + 2(30+1) =

10·30 +10·1 +2·30 + 2·1 =

300 + 10 +60 + 2 = 372

	30	1
10	300	10
2	60	2

c. (25) (60) = 1500

(20+5)(60+0) =

20(60) + 5(60) =

1200 + 300 = 1500

	60
20	1200
5	300

d. (134) (21) = 2814

(100 +30 +4)(20 +1) =

(100)(20+1) + 30(20+1) +4(20+1) =

100·20 +100·1 +30·20 +30·1 +4·20 +4·1 =

2000 +100 +600 +30 +80 +4 =

2814

	20	1
100	2000	100
30	600	30
4	80	4

e. (436) (107) = 46,652

(400+30 +6)(100 +7) =

400(100+7) +30(100+7) +6(100+7) =

400·100 +400·7 +30·100 +30·7 +6·100 +6·7 =

40,000 + 2,800 + 3,000 + 210 + 600 + 42 =

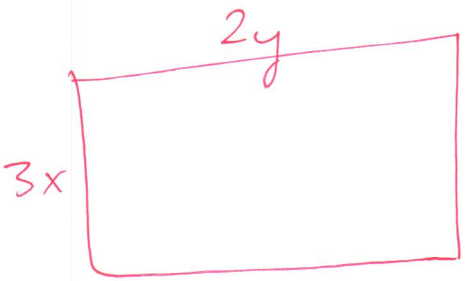
46,652

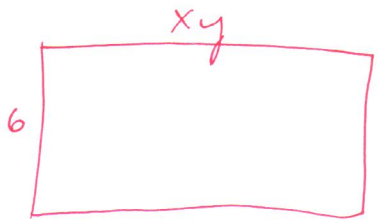
	400	30	6
100	40000	3000	600
7	2800	210	42

3. Draw a picture to represent the following products:

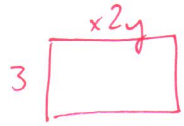
   a. $(3x)(2y)$

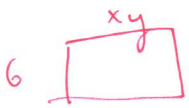
   b. $6xy$

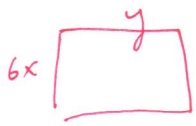




other possible answers (if the multiplication is grouped differently)







etc.

SUMMARY (What I learned in this section)