FACTORS AND MULTIPLES 3

Name: Key Date:_____ Period:_____

SECTION 3.2 EXPONENTS AND ORDER OF OPERATIONS

VOCABULARY

DEFINITION	EXAMPLE
Exponential Notation: expresses a number in terms of a base and an exponent	$9 = 3^{2}$ (2 ← exponent, 3 ← base)
Power or exponent: The number of times the base is multiplied	3^{2} ←
Base: The number that is raised to a power (multiplied repeatedly)	→ 3^{2}
Order of Operations: The order mathematical operations are performed: Parentheses, Exponents, Multiplication/Division, Addition/Subtraction	P (x+1) E 3^{2} M D > left to right A S > left to right

Big Idea: How do you use order of operations to compute numerical expressions?

EXPLORATION 1: EXPONENTIAL GROWTH

Your uncle is asking you what you would prefer for your birthday gift, and gives you a choice of $50 every year for your birthday or $1 for this birthday and double the amount every year thereafter.

Fill in the table below to compare the outcome of the two choices.

Year	1	2	3	4	5	6	7	8	9	10	11	12	13
Choice 1: $50 per year	$50	$50	$50	$50	$50	$50	$50	$50	$50	$50	$50	$50	$50
Choice 2: $1 doubling each year	$1	$2	$4	$8	$16	$32	$64	$128	$256	$512	$1024	$2048	$4096

a. After 13 years, what is the total amount you would have received if you selected Choice 1? $650

b. After 13 years, what is the total amount you would have received if you selected Choice 2? $8191

EXPLORATION 2: REPEATED MULTIPLICATION

Consider the problem 2 + 2 + 2 + 2 + 2 = 10. We know that we can write this problem more simply using multiplication as 2 · 5 = 10. Now consider 2 · 2 · 2 · 2 · 2 = 32 or 2^5. Just as multiplication takes the place of repeated addition, we can use a base and an exponent to indicate repeated multiplication, where the base is the repeated factor and the exponent, or power, tells how many times the base is multiplied by itself, as illustrated below.

EXPONENTIAL FORM	EXPANDED FORM	STANDARD FORM
base → 2^5 ← exponent	$= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$= 32$

EXPONENTIAL FORM	EXPANDED FORM	STANDARD FORM
7^2	7·7	49
6^3	6 · 6 · 6	216
7^3 (or 243)	7·7·7 (or 243)	243
1^7	1 · 1 · 1 · 1 · 1 · 1 · 1	1
10^6	10·10·10·10·10·10	1,000,000
4^5	4 · 4 · 4 · 4 · 4	1024
5^2 (or 25)	5·5 (or 25)	25
11^3	11 · 11 · 11	1331
2^8	2·2·2·2·2·2·2·2	256
8^1	8	8

EXPLORATION 3: MULTIPLICATION OF POWERS

By using the definition of exponential notation and multiplication, we see that:

$$3^4 \cdot 3^6 = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) = 3^{10} = 3^{4+6}.$$

Compute the following products, showing all your work.

a. $3^2 \cdot 3^3 = (3 \cdot 3)(3 \cdot 3 \cdot 3) = 3^{2+3} = 3^5$ b. $3^3 \cdot 3^2 = (3 \cdot 3 \cdot 3)(3 \cdot 3) = 3^{3+2} = 3^5$

c. $2^4 \cdot 2^2 = (2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2) = 2^{4+2} = 2^6$ d. $10^3 \cdot 10^5 = (10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) = 10^{3+5} = 10^8$

What pattern do you observe when multiplying numbers in exponential form with the same base? Explain.

The exponent of the product is the sum of the exponents of the products.

EXPLORATION 4: SPECIAL CASES

What do 4^1 and 4^0 equal?

We note that $4 \cdot 4 = 4^2 = 4^{1+1} = 4^1 \cdot 4^1$, so 4^1 must be the same as 4. We can use the same process for any number x: $x \cdot x = x^2 = x^{1+1} = x^1 \cdot x^1$, so $x^1 = x$.

What does 4^0 equal? Because $4 \cdot 4^0 = 4^1 \cdot 4^0 = 4^{1+0} = 4^1 = 4 = 4 \cdot 1$, we see that multiplying by 4^0 is the same as multiplying by the number 1. We, therefore, assume that for any positive integer n, $n^0 = 1$.

Consider the number 4638, which we read as four thousand six hundred thirty eight. Using place value and our notation of exponents, we can rewrite 4638 using expanded notation in the following way:

$$4 \cdot 1000 + 6 \cdot 100 + 3 \cdot 10 + 8 \cdot 1 = 4 \cdot 10^3 + 6 \cdot 10^2 + 3 \cdot 10^1 + 8 \cdot 10^0$$

Start with the expression $4 \cdot 10^3 + 6 \cdot 10^2 + 3 \cdot 10^1 + 8 \cdot 10^0$, or in calculator notation, $4 \times 10^3 + 6 \times 10^2 + 3 \times 10^1 + 8 \times 10^0$. In what order can we perform the calculations in this expression so the sum equals 4638?

To summarize, any number to the first power is equal to the base and any non-zero number raised to the power of zero is equal to 1. Every time!

Rewrite each of the following in standard form.

1. $4^1 =$ 4
2. $7^0 =$ 1
3. $n^1 =$ n
4. $5^1 + 4^0 =$ 5+1 = 6
5. $x^1 - y^0 =$ x-1

EXPLORATION 5: ORDER OF OPERATIONS

Compute the following, showing all your work.

$20 - 10 \div 2 + 3^3 - 9$

$20 - 5 + 27 - 9$

$15 + 27 - 9$

$42 - 9$

33

student attempts may vary this time, but should be correct on page 93.

If possible, check your work on a calculator. (To enter 3^3 on a calculator, we enter 3^3). Would you be surprised to know that the answer is not 9? The order that a calculator uses is called the Order of Operations.

We summarize below the order in which mathematical operations are performed:

Order of Operations

Compute the numbers inside the parentheses or grouping symbols.

Compute any exponential expressions.

Multiply or divide as they occur from left to right.

Add or subtract as they occur from left to right.

Why do these two problems have different solutions? grouping symbols change what order the steps should be done.

a. $7 \cdot 8 - 6 \div 2$

$56 - 3$

53

b. $7 \cdot (8 - 6) \div 2$

$7 \cdot (2) \div 2$

$14 \div 2$

7

Let's try our first problem again using the correct order. Carry out one operation for each step and write your result. Continue this process until you are able to find the one number that equals the numerical expression given. Add more steps if you need them.

$20 - 10 \div 2 + 3^3 - 9$ $= 20 - 5 + 3^3 - 9$

$= 20 - 5 + 27 - 9$

$= 15 + 27 - 9$

$= 42 - 9$

$= 33$

EXPLORATION 6: MORE ORDER OF OPERATIONS

PEMDAS or GEMS

Use the table as a guide and work your problem in the right-hand column.

PEMDAS		GEMS		$4 + 2^3 \cdot 3 - (17\text{-}5) \cdot 2 + (17\text{-}5) \div 2$
P	Parenthesis ( )	**G**	Grouping ( ), [ ], { }, | |	$4+2^3 \cdot 3 - (12) \cdot 2 + (12) \div 2$
E	Exponents x^y	**E**	Exponents x^y	$4+8 \cdot 3 - 12 \cdot 2 + 12 \div 2$
MD	Multiply **OR** Divide, Whichever comes first from left to right	**M(D)**	Multiply **OR** Divide, Whichever comes first from left to right	$4 + 24 - 24 + 6$
AS	Add **OR** Subtract, Whichever comes first from left to right	**S(A)**	Add **OR** Subtract, Whichever comes first from left to right	$28 - 24 + 6$ $4 + 6$ 10

PROBLEMS:

1. Rewrite each of the following in exponential form.

   a. $18 \cdot 18 \cdot 18 \cdot 18 \cdot 18 \cdot 18 \cdot 18 \cdot 18 =$ 18^8

   b. $(4 \cdot 4 \cdot 4 \cdot 4) + (87 \cdot 87) =$ $4^4 + 87^2$

   c. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 =$ $5^4 \cdot 7^5$

   d. $n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n \cdot n =$ n^{10}

   e. $5 =$ 5^1

   f. $s \cdot s \cdot s \cdot b \cdot b =$ $s^3 b^2$

2. Expand and compute the answer of the following. Tell which expression is greater or if they are equal. For example,

   4^3 or $3^4 = 4 \cdot 4 \cdot 4$ or $3 \cdot 3 \cdot 3 \cdot 3$. Then $64 < 81$

   a. 2^3 or $4^2 =$ $2 \cdot 2 \cdot 2$ or $4 \cdot 4$. Then $8 < 16$

   b. 5^3 or $3^5 =$ $5 \cdot 5 \cdot 5$ or $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. Then $125 < 243$

   c. 1^6 or $4^0 =$ $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$ or $4^0 = 1$. Then $1 = 1$

3. Evaluate the following expressions by expanding then writing your solution.

   a. $4^2 + 8 =$ $4 \cdot 4 + 8 = 16 + 8 =$ 24

   b. $3^3 + 5^2 =$ $3 \cdot 3 \cdot 3 + 5 \cdot 5 = 27 + 25 =$ 52

   c. $3^4 + 4^3 =$ $3 \cdot 3 \cdot 3 \cdot 3 + 4 \cdot 4 \cdot 4 = 81 + 64 =$ 145

4. Evaluate the following numerical expressions using Order of Operations:

   a. $7 + (5 - 2)^3 - 16 \div 2$

   $7 + (3)^3 - 8$

   $7 + 27 - 8$

   26

   b. $(81 \div 9)^2 \cdot (27 - 24)$

   $(9)^2 \cdot (3)$

   $81 \cdot 3$

   243

   c. $24 \div 8 \cdot 7 - 1^4$

   $3 \cdot 7 - 1$

   $21 - 1$

   20

d. $9 - 5 \div (8 - 3) \cdot 2 + 4$

$9 - 5 \div (5) \cdot 2 + 4$

$9 - 1 \cdot 2 + 4$

$9 - 2 + 4 = 7 + 4$

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e. $16 - 3(8 - 3)^2 \div 5$

$16 - 3(5)^2 \div 5$

$16 - 3 \cdot 25 \div 5$

$16 - 75 \div 5$

$16 - 15 = 1$

f. $7 + (6 \cdot 5^2 - 4^2)$

$7 + (6 \cdot 25 - 16)$

$7 + (150 - 16)$

$7 + 134 = 141$

5. Escherichia coli bacteria are more commonly known as E. Coli. A scientist places one of the living bacteria in a petri dish. The number of bacteria in the dish doubles each hour.

   a. How many bacteria are in the dish after 1 hour? 2

   b. How many bacteria are in the dish after 3 hours? 8

   c. How many bacteria are in the dish after 5 hours? 32

   d. How many bacteria are in the dish after n hours? 2^n

SUMMARY (What I learned in this section)