FACTORS AND MULTIPLES 3

Name: Key Date:______ Period:______

SECTION 3.3 PRIME FACTORIZATION

VOCABULARY

DEFINITION	EXAMPLE
Prime Factorization: process or result of finding the prime factors of a number	$24 = 2^3 \cdot 3$
Perfect Square: an integer n that can be written in the form $n = k^2$ where k is an integer	$\rightarrow 9 = 3^2$ $\rightarrow 25 = 5^2$
Perfect Cube: an integer n that can be written in the form $n = k^3$ where k is an integer	$\rightarrow 8 = 2^3$ $\rightarrow 27 = 3^3$
Fundamental Theorem of Arithmetic: if n is an integer $n > 1$ then n is prime OR can be written as the product of primes $p_1, p_2, \ldots, p_k$ where $p_1 \leq p_2 \ldots \leq p_k$	$24 = 2 \cdot 2 \cdot 2 \cdot 3$ $2 \leq 2 \leq 2 \leq 3$

and k is a natural number. There is only 1 way to write n in this form.

Big Idea: How do you find the prime factorization of numbers?

EXPLORATION 1: FACTORING A NUMBER AS PRODUCT OF PRIMES

Consider the number 60. What number pairs can we multiply together to get 60? Make a list of the factor pairs.

1×60, 2×30, 3×20, 4×15, 5×12, 6×10

Now, let's choose just one of the factor pairs. For example, $4 \cdot 15 = 60$. Are either of these numbers prime? No! Let's find the factors *of the factors.*

$4 = 2 \cdot 2$ and $15 = 3 \cdot 5$

Looking at $2 \cdot 2$ and $3 \cdot 5$, we see that all of these numbers are prime. (It may be helpful to refer back to your Sieve of Eratosthenes that you created in Section 3.1.) Therefore, the prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$.

It's fun to factor a number to the product of prime numbers, and easy to check your work. Find the product of $2 \cdot 2 \cdot 3 \cdot 5$, and verify that it is 60.

If you had chosen a different factor pair to begin with, would your final answer be different? Pick another factor pair for 60 and check.

No, the prime factorization of 60 is always the same.

2×30	3×20	5×12	6×10
$2 \times 3 \times 10$	$3 \times 5 \times 4$	$5 \times 2 \times 6$	$2 \times 3 \times 10$
$2 \times 3 \times 2 \times 5$	$3 \times 5 \times 2 \times 2$	$5 \times 2 \times 3 \times 2$	$2 \times 3 \times 2 \times 5$
$2 \cdot 2 \cdot 3 \cdot 5$	$2 \cdot 2 \cdot 3 \cdot 5$	$2 \cdot 2 \cdot 3 \cdot 5$	$2 \cdot 2 \cdot 3 \cdot 5$

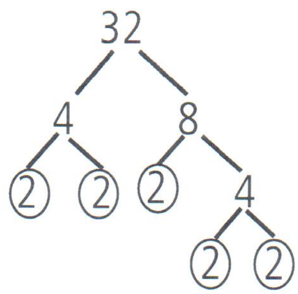
Now try to write 24 as a product of prime numbers: $2 \cdot 2 \cdot 2 \cdot 3$ $(2^3 \cdot 3)$

Don't forget to check your work!

$$24 = 4 \times 6$$
$$= (2 \times 2) \times (2 \times 3)$$

EXPLORATION 2: FACTOR TREES

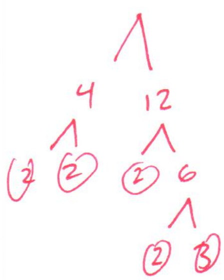
A useful way to organize your work is through a **tree diagram**, or **factor tree**.



In the example above, 32 is factored to 4 · 8 in the first branch of the factor tree. Since neither 4 nor 8 are prime, the branches will continue until we reach only prime numbers. One reason we are so interested in prime numbers is that they are the building blocks of the integers. In the previous section, we learned that a prime number is a positive integer *greater than 1* that can be written as a product of only two positive integers. Keep in mind that 1 is *not* a prime number. Therefore, it should not be included in your factor tree.

Find the prime factorization of the following numbers using a factor tree.

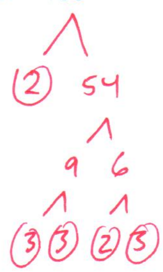
a. 48



$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

$2^4 \cdot 3$

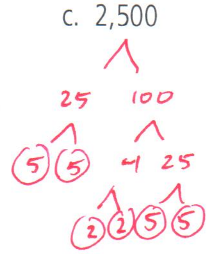
b. 108



$2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

$2^2 \cdot 3^3$

c. 2,500



$2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 5$

$2^2 \cdot 5^4$

EXPLORATION 3: PRIME FACTORIZATION CHART

Use any method you choose to prime factor the integers 1 – 100. Write the prime factorization in exponential form, as shown in the examples already done for you.

1	Not prime or composite!	21	$3 \cdot 7$	41	41	61	61	81	3^4
2	2	22	$2 \cdot 11$	42	$2 \cdot 3 \cdot 7$	62	$2 \cdot 31$	82	$2 \cdot 41$
3	3	23	23	43	43	63	$3^2 \cdot 7$	83	83
4	$2 \cdot 2$	24	$2^3 \cdot 3$	44	$2^2 \cdot 11$	64	2^6	84	$2^2 \cdot 3 \cdot 7$
5	5	25	5^2	45	$3^2 \cdot 5$	65	$5 \cdot 13$	85	$5 \cdot 17$
6	$2 \cdot 3$	26	$2 \cdot 13$	46	$2 \cdot 23$	66	$2 \cdot 3 \cdot 11$	86	$2 \cdot 43$
7	7	27	3^3	47	47	67	67	87	$3 \cdot 29$
8	2^3	28	$2^2 \cdot 7$	48	$2^4 \cdot 3$	68	$2^2 \cdot 17$	88	$2^3 \cdot 11$
9	3^2	29	29	49	7^2	69	$3 \cdot 23$	89	89
10	$2 \cdot 5$	30	$2 \cdot 3 \cdot 5$	50	$2 \cdot 5^2$	70	$2 \cdot 5 \cdot 7$	90	$2 \cdot 3^2 \cdot 5$
11	11	31	31	51	$3 \cdot 17$	71	71	91	$7 \cdot 13$
12	$2^2 \cdot 3$	32	2^5	52	$2^2 \cdot 13$	72	$2^3 \cdot 3^2$	92	$2^2 \cdot 23$
13	13	33	$3 \cdot 11$	53	53	73	73	93	$3 \cdot 31$
14	$2 \cdot 7$	34	$2 \cdot 17$	54	$2 \cdot 3^3$	74	$2 \cdot 37$	94	$2 \cdot 47$
15	$3 \cdot 5$	35	$5 \cdot 7$	55	$5 \cdot 11$	75	$3 \cdot 5^2$	95	$5 \cdot 19$
16	2^4	36	$2^2 \cdot 3^2$	56	$2^3 \cdot 7$	76	$2^2 \cdot 19$	96	$2^5 \cdot 3$
17	17	37	37	57	$3 \cdot 19$	77	$7 \cdot 11$	97	97
18	$2 \cdot 3^2$	38	$2 \cdot 19$	58	$2 \cdot 29$	78	$2 \cdot 3 \cdot 13$	98	$2 \cdot 7^2$
19	19	39	$3 \cdot 13$	59	59	79	79	99	$3^2 \cdot 11$
20	$2^2 \cdot 5$	40	$2^3 \cdot 5$	60	$2^2 \cdot 3 \cdot 5$	80	$2^4 \cdot 5$	100	$2^2 \cdot 5^2$

EXPLORATION 4: PERFECT CUBES

A **perfect cube** is an integer n that can be written in the form $n = k^3$, where k is an integer. Some examples of perfect cubes are

$$0^3 = 0,\ 1^3 = 1,\ 2^3 = 8,\ 3^3 = 27,\ 4^3 = 64,\ \ldots.$$

How can you use the prime factors of a number to determine whether it is a perfect cube?

Answers may vary slightly: every prime factor must have an exponent that is a multiple of 3.

PROBLEMS:

1. Determine as efficiently as possible whether each of the following numbers is prime or composite. Prove your answer with a factor pair if you believe the number is composite.

   a. 51 3×17

   b. 235 5×47

   c. 159 3×53

   d. 119 7×17

   e. 31 prime

   f. 790 10×79

2. Write the prime factorizations of 12 and 24. Looking at your factorizations, explain what the answers have in common and what are their main differences.

   $12 = 2^2 \cdot 3$ $24 = 2^3 \cdot 3$

   They share all of the factors of 12, and the prime factorization of 24 has one more 2.

3. Write the prime factorizations of 8 and 49 in exponential form. Looking at your factorizations, explain what the answers have in common and what are their main differences.

   $8 = 2^3$ $49 = 7^2$

   Each has a different base and exponent, but both have only 1 prime factor.

SUMMARY (What I learned in this section)