FACTORS AND MULTIPLES 3

Name: Key **Date:**_______ **Period:**______

SECTION 3.4 Common Factors and the GCF

VOCABULARY

DEFINITION	EXAMPLE
Common Factor: a factor that two or more integers have in common	$6 = 2 \cdot 3$ $18 = 2 \cdot 3 \cdot 3$ 2 or 3 or 6
Greatest Common Factor: the greatest positive integer that is a factor of both m and n	GCF(6, 18) = 6 [see above]
Relatively Prime: two numbers are relatively prime if their greatest common factor is 1	GCF(15, 8) = 1 15 & 8 are relatively prime

Big Idea: How do you find the greatest common factor of two or more numbers?

EXPLORATION 1: FERNANDO AND THE FROG-JUMPING CONTEST

Pick three different colors to represent the 1-frog, 2-frog, and 3-frog in the activity that follows.

To prepare for a frog-jumping contest, Fernando decided to train a group of his fellow frogs. Each frog was trained to jump a certain length along a number line starting at 0. He trained a 1-frog to jump a distance of 1 unit in each hop. He also trained a 2-frog to jump 2 units, a 3-frog to jump 3 units and so on. The frogs always start at the zero point on the number line. Now Fernando wants to know which frogs will land on certain locations on the number line. (Use the number line provided to answer the following questions.)

1. Which of his frogs will land on both the locations 24 and 36? 1, 2, 3, 4, 6, 12

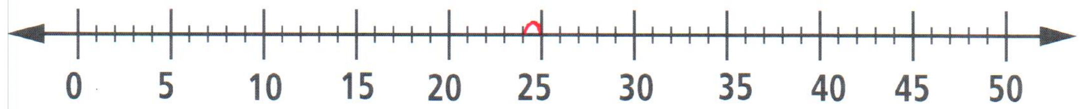
2. Which is the longest jumping frog that will land on both 24 and 36? Explain why this answer makes sense.

   The 12-frog, because it is the largest divisor that 24 and 36 have in common.

   $12 \times 2 = 24$ $12 \times 3 = 36$

3. What is the longest jumping frog that will land on both 20 and 32? A 4-frog.

$4 \times 5 = 20$
$4 \times 8 = 32$



4. What is the longest jumping frog that will land on both 24 and 25? 1-frog.
Use the number line above to explain.

EXPLORATION 2: IDENTICAL STRINGS

Suppose we have different lengths of two types of string. The cotton string is 120 inches long, and the nylon string is 72 inches long. Determine every possible integer length both of the strings can be cut so that each piece is the same length. Make a list of all the possible common lengths. What is the longest common length possible? In this Exploration, each piece must be cut into a positive integer length with no fractions and no string left.

The longest common length possible is: 24 inches

120	
1	120
2	60
3	40
4	30
5	24 *
6	20
8	15
10	12

72	
1	72
2	36
3	24 *
4	18
6	12
8	9

EXAMPLE 1:

There are several different ways to calculate the GCF of two numbers. Here is one way that reinforces the term **G**reatest **C**ommon **F**actor. Find the GCF of 15 and 25 using a Factor T-Chart listing all of the factor pairs.

15	
1	15
3	5

25	
1	25
5	5

List all the factors 15 and 25 have in common. 1, 5

What is the greatest factor these two numbers have in common? 5

EXAMPLE 2:

Find the GCF of 27 and 32.

27	
1	27
3	9

32	
1	32
2	16
4	8

List all the factors 27 and 32 have in common. 1

What is the greatest factor these two numbers have in common? 1

These integers are examples of relatively prime numbers. In your own words, write a definition of relatively prime numbers.

The GCF of relatively prime numbers is 1. The greatest factor that both relatively prime numbers share is 1.

From the definition above, the numbers 27 and 32 are relatively prime. Notice that neither 27 nor 32 are prime numbers. If we consider two prime numbers like 3 and 7, what is their GCF? Check a few more examples. Make a generalization about the GCF of any two prime numbers.

GCF(3, 7) = 1. GCF(19, 5) = 1. GCF(13, 2) = 1. (examples checked will vary.) The GCF of any two prime numbers is 1.

EXAMPLE 3:

Although Factor T-Charts are effective, they are not as efficient with larger numbers. Consider the integers 108 and 168 using the process of first finding the factors, then the common factors, and finally the GCF.

First, list all the factors of 108. Then, list all the factors of 168. Make sure you have 12 factors for 108 and 16 factors for 168. There are many factors to find. If you do not have them all listed, go back and find them. Next, determine all the factors the two numbers have in common.

d. GCF of 80 and 64 = 16

$80 = 2^4 \cdot 5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

$64 = 2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

e. GCF of 50 and 35 = 5

$50 = 2 \cdot 5^2 = 2 \cdot 5 \cdot 5$

$35 = 5 \cdot 7 = 5 \cdot 7$

f. GCF of 24 and 18 = 6 $= 2 \cdot 3$

$24 = 2^3 \cdot 3 = 2 \cdot 2 \cdot 2 \cdot 3$

$18 = 2 \cdot 3^2 = 2 \cdot 3 \cdot 3$

2. In each part of this exercise, the prime factorizations of two numbers are given. First, use the prime factorizations to find the GCF of the two numbers then, compute (find the value of) the two numbers from their prime factors.

a. $2 \cdot 3 \cdot 7$

$2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$

GCF = 42

Values: 42, 630

b. $2^3 \cdot 3^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

$2 \cdot 3^3 = 2 \cdot 3 \cdot 3 \cdot 3$

GCF = $2 \cdot 3^2 = 18$

Values: 72, 54

c. $5 \cdot 7^2 = 5 \cdot 7 \cdot 7$

$2 \cdot 7^3 = 2 \cdot 7 \cdot 7 \cdot 7$

GCF = 49

Values: 245, 686

3. For each pair of integers below, find the GCF of the two integers using prime factorization.

a. 24 and 62

24 → 3, 8; 8 → 2, 4; 4 → 2, 2

62 → 2, 31

b. 115 and 225

115 → 5, 23

225 → 5, 45; 45 → 15, 3; 15 → 3, 5

c. 79 and 83

79 (prime)

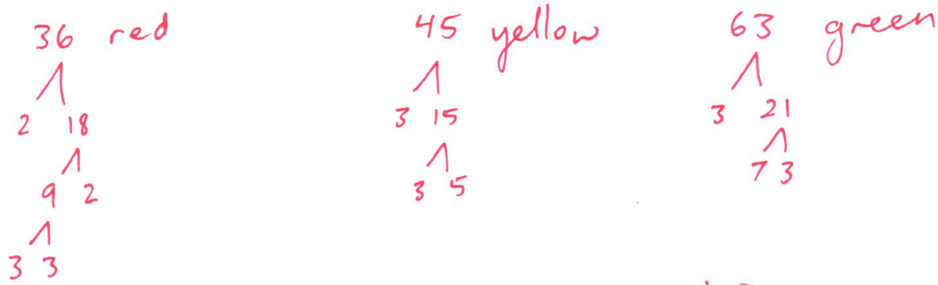
83 (prime)

GCF of 24 and 62 = 2

GCF of 115 and 225 = 5

GCF of 79 and 83 = 1

4. Sarah has 36 red beads, 45 yellow beads, and 63 green beads. She wants to separate them into identical groups to make bracelets for her friends. What is the greatest number of bracelets she can make so that each bracelet has the same number of each color of bead and there are no beads left over?

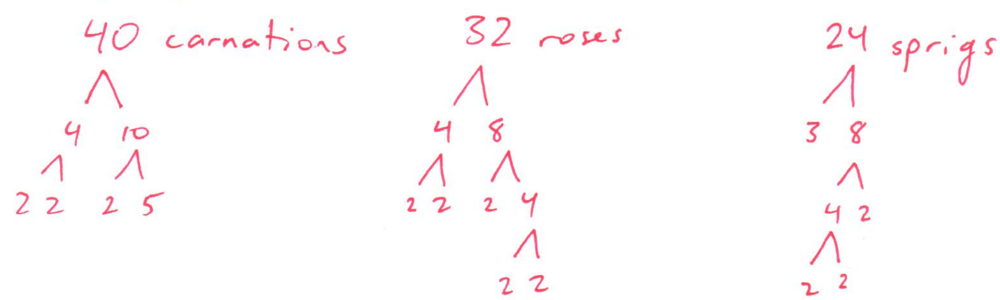


$2 \cdot 2 \cdot 3 \cdot 3$ $\qquad$ $3 \cdot 3 \cdot 5$ $\qquad$ $3 \cdot 3 \cdot 7$

Sarah can make ___9___ bracelets.

Each bracelet will have ___4___ red, ___5___ yellow, and ___7___ green beads.

5. LiiLii has 40 carnations, 32 roses, and 24 sprigs of baby's breath. She is making flower arrangements to set at the tables in her diner. What is the greatest number of vases she can fill with matching arrangements?



$2^3 \cdot 5$ $\qquad$ 2^5 $\qquad$ $2^3 \cdot 3$

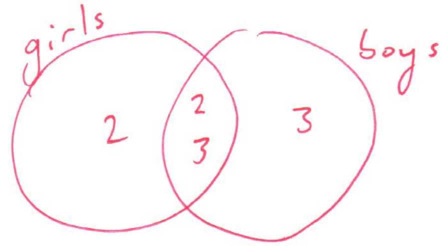
$2 \cdot 2 \cdot 2 \cdot 5$ $\qquad$ $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ $\qquad$ $2 \cdot 2 \cdot 2 \cdot 3$

LiiLii can make ___8___ arrangements.

Each arrangement will have ___5___ carnations, ___4___ roses, and ___3___ sprigs of baby's breath.

6. There are 12 girls and 18 boys in Ms. Girardeau's math class. What is the greatest number of identical groups Ms. Girardeau can make so that no children are left out?

$12 = 2^2 \cdot 3$ $18 = 2 \cdot 3^2$



Ms. Girardeau can make <u>6</u> groups with <u>3</u> boys and <u>2</u> girls in each group.

7. Make a list of any keywords or phrases that indicate finding the Greatest Common Factor.

"greatest common" (Answers may vary.)

"GCF"

"most groups with the same number of 2 or more things in each"

SUMMARY (What I learned in this section)