FACTORS AND MULTIPLES 3

Name: Key Date:_____ Period:_____

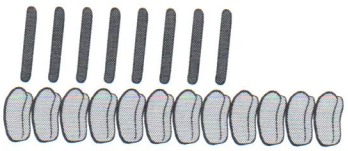
SECTION 3.5 COMMON MULTIPLES AND THE LCM

VOCABULARY

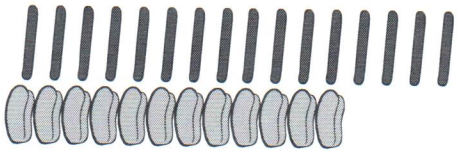
DEFINITION	EXAMPLE
Common Multiple: m is a common multiple of a and b if m is a multiple of both a and b.	48 is a common multiple of 4 and 24
Least Common Multiple: The smallest positive integer that is a common multiple	LCM(4, 24) = 24

Big Idea: How do you find the least common multiple of two or more numbers?

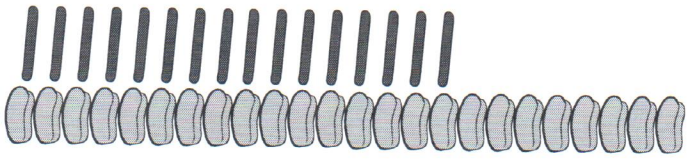
Have you noticed that hot dogs often come in packages of eight, and hot dog buns come in packages of twelve? When people plan to cook hot dogs, they tend to buy one package of hot dogs and one package of buns. But if they do this, they are left with four extra buns.



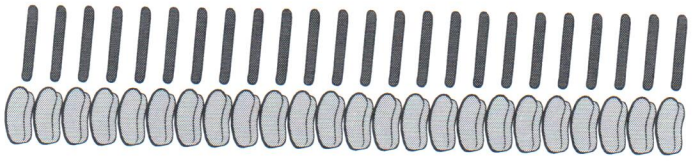
Some people who pay for the extra hot dog buns don't want to waste them. What can they do? They could buy another package of eight hot dogs:



But now there are four extra hot dogs without buns. If they buy more buns:



There are eight buns without hot dogs, even more extra buns than the first time. Will this process ever end? Try buying one more package of hot dogs:



Aha! We have finally reached a point where we have exactly the same number of hot dogs and buns. Of course, in order to get there, the consumers had to buy two packages of buns and three packages of hot dogs. Maybe they can freeze the rest.

What happened mathematically with the hot dogs and buns? One way to organize the number of hot dogs and the number of buns is to create a table. Fill in the missing cells on the table.

# Packages	1	2	3	4	5	6	7	8
Hot dogs	8	16	24	32	40	48	56	64
Buns	12	24	36	48	60	72	84	96

Because you want to buy the least amount possible, you look through your table to find the smallest number they each have in common. Then determine how many packages of hot dogs and hot dog buns you will need to buy in order to have the same amount of each.

The smallest number they each have in common is 24. Therefore, you should buy 3 packages of hot dogs and 2 packages of hot dog buns.

Notice that 48 and 72 are also common multiples of 8 and 12 but not the least.

EXPLORATION 2: COMMERCIAL TIME

Radio station KISS broadcasts a commercial every 22 minutes. WILD broadcasts a commercial every 12 minutes. If the two stations broadcast their commercials at 3:20 , when is the next time their commercials will air at the same time?

To solve this problem we begin by listing the multiples of 22 and 12. You can organize this on a table. List the first 10 multiples in the table below.

WILD	12	24	36	48	60	72	84	96	108	120
KISS	22	44	66	88	110	132	154	176	198	220

Do you see a common multiple? Sometimes it is necessary to continue skip counting. Since 12 is the smaller number, let's extend our list of multiples further.

What are the next two multiples of 12? 132, 144

By now, you should see a common multiple. What is the LCM of 12 and 22? 132

Looking back at our original problem, we see that we not only need to find the LCM, but also the next time the commercials will be playing simultaneously. Remember, they aired together last at 3:20. Use the LCM to determine your answer.

The commercials will air together again at 5:32.

132 min + 3 hr + 20 min

3 hr + 152 min

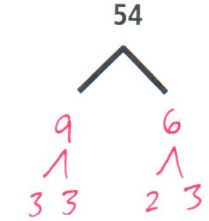
3 hr + 120 min + 32 min

3 hr + 2 hr + 32 min

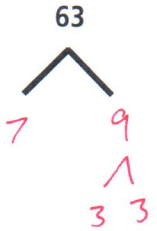
EXAMPLE 1:

As we found with the GCF, there are several different ways to calculate the LCM of two numbers. Once again, we'll see that prime factorization can make this process simpler.

Let's try finding the LCM of 54 and 63 using prime factorization. Begin by using a factor tree to find the prime factorization of 54 and 63.



$2 \cdot 3^3$



$3^2 \cdot 7$

List the prime factors of each integer from least to greatest, stacking the common prime factors:

54: $2 \cdot 3 \cdot 3 \cdot 3$

63: $3 \cdot 3 \cdot 7$

Now look at the method for finding the LCM of numbers using their prime factorization. The factors of each are as follows: 54: $3 \cdot 3 \cdot 3 \cdot 2$ and 63: $3 \cdot 3 \cdot 7$. By examining the two sets of prime factors, you can see that a common multiple must include 2, 3, and 7. However, $2 \cdot 3 \cdot 7 = 42$ is not a multiple of 54 or 63. Because 54 has three factors of 3 and 63 has two factors of 3, to include both numbers, use three factors of 3. Why won't two factors of 3 be enough? Now multiply the factors 2, 3^3, and 7.

The number we're looking for is a multiple of three factors of 3. $2 \cdot 3^3 \cdot 7 =$ 378

This is the smallest integer that contains all of the building blocks, or prime factors, in both sets of prime factors. Therefore, 378 is the LCM of 54 and 63.

Now find the LCM of each pair of numbers below using the prime factorization method:

a. 36 and 42

Prime Factor Lists

36: $2 \cdot 2 \cdot 3 \cdot 3$

42: $2 \cdot 3 \cdot 7$

LCM: $252 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$

b. 17 and 18

Relatively prime. 17 and $2 \cdot 3^2$

$2 \cdot 3 \cdot 3 \cdot 17 = 306$ is the LCM.

18
2 9
3 3

c. 13 and 52

13 and 52

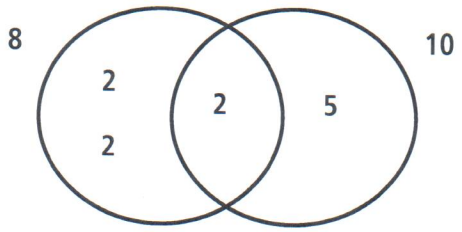
13 = 13

52 = 2 · 2 · 13

LCM is 52.

EXPLORATION 3: FINDING THE LCM WITH A VENN DIAGRAM

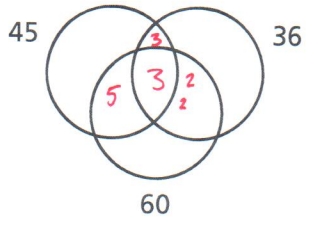
Another approach to solving for the LCM is to look at the Venn diagram, as you did in Section 3.4, when you worked with GCFs. Examine the prime factors of 8 and 10. The Venn diagram includes the prime factors for each number in the respective circles. Note the common factor of 2 in the overlapping part of the circles. 2 is the GCF. What must the LCM be?



You will find that the Venn diagram is more practical when the numbers are larger and there are three numbers or more. For example, to find the least common multiple of 45, 36, and 60, write all the prime factors and notice which factors are common.

$$45 = 3^2 \cdot 5$$
$$36 = 2^2 \cdot 3^2$$
$$60 = 2^2 \cdot 3 \cdot 5$$

Use the Venn diagram to represent this information.



After you have separated the factors into different regions in the Venn diagram by multiplying all the numbers in the circles and their intersections you will have the LCM of 45, 36, and 60. Show your work below.

$3 \cdot 3 \cdot 5 \cdot 2 \cdot 2 = 180$

The LCM of 45, 36, and 60 is 180.

PROBLEMS:

1. Find the Least Common Multiple of each pair of numbers by listing multiples of the two numbers until you find the first multiple common to both lists.

   a. 6 and 9, LCM = 18

   6: 6, 12, (18)
   9: 9, (18)

   b. 15 and 18, LCM = 90

   15: 15, 30, 45, 60, 75, (90) 105
   18: 18, 36, 54, 72, (90)

   c. 8 and 12, LCM = 24

   8: 8, 16, (24)
   12: 12, (24)

2. In each part of this exercise, the prime factorizations of two numbers are given. First, use the prime factorizations to find the LCM of the two numbers then, compute (find the value of) the two numbers from their prime factors.

   a. $2 \cdot 3 \cdot 5$

   $2 \cdot 5 \cdot 7 \cdot 9$

   LCM = $2 \cdot 5 \cdot 3 \cdot 7 \cdot 9 = (1890)$

   Values: 30, 630

   b. $2^2 \cdot 3^2 = 2 \cdot 2 \cdot 3 \cdot 3$

   $2 \cdot 3^3 = 2 \cdot 3 \cdot 3 \cdot 3$

   LCM = $2 \cdot 3 \cdot 3 \cdot 2 \cdot 3 = (108)$

   Values: 36, 54

   c. $2 \cdot 5 \cdot 7^2$

   $2 \cdot 7^3$

   LCM = $2 \cdot 7^2 \cdot 5 \cdot 7 = (3430)$

   Values: 490, 686

3. For each pair of integers below, find the LCM of the two integers using prime factorization.

a. 24 and 62

2 · 12 ; 2 · 31 ; 2 · 6 ; 2 · 3

LCM = $2 \cdot 2 \cdot 2 \cdot 3 \cdot 31 =$ 744

b. 115 and 225

5 · 23 ; 5 · 45 ; 15 · 3 ; 3 · 5

LCM = $5 \cdot 23 \cdot 3 \cdot 3 \cdot 5 =$ 5175

c. 79 and 83

prime ; prime

LCM = $79 \cdot 83 =$ 6557

4. Brian and Lydia are running laps around the track. Brian can run one lap in 7 minutes, and Lydia can run a lap in 3 minutes. If they start together, how many minutes will pass before they are crossing the start line together again?

B 7 14 (21)

L 3 6 9 12 15 18 (21)

OR 3 and 7 have no factors in common so their LCM is 3·7.

Brian and Lydia will cross the starting line together again in 21 minutes.

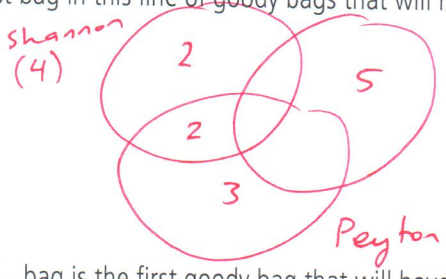
5. The red shuttle bus at Texas State University stops at every fifth building on University Drive. The blue shuttle bus stops at every fourth building on this route. If they leave the first building together, what building will they both stop at next?

Red 5 10 15 (20)

Blue 4 8 12 16 (20)

The red and blue shuttle buses will both stop at the 20th building next.

6. In a long line of goody bags, Shannon is placing a special prize in every fourth goody bag. Colton is placing a different prize in every fifth bag. Peyton is placing another prize in every sixth bag. What is the first bag in this line of goody bags that will have all three prizes?



$2 \cdot 2 \cdot 3 \cdot 5 = 60$

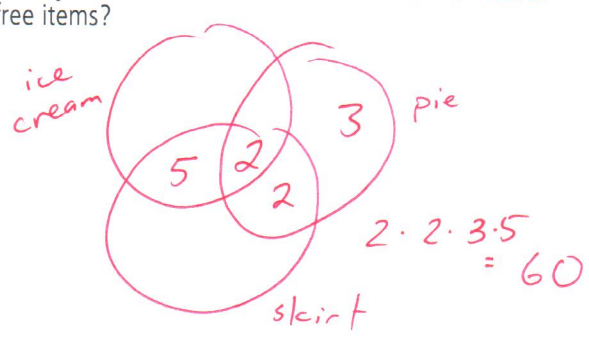
The 60th bag is the first goody bag that will have all three prizes.

7. On its opening night, Hilary's Hula House is giving every tenth customer a free ice cream cone, every twelfth customer a free slice of pie, and every twentieth customer a free Hula skirt. Which customer will be the first to receive all three free items?

ice cream $10 = 2 \cdot 5$

pie $12 = 2 \cdot 2 \cdot 3$

skirt $20 = 2 \cdot 2 \cdot 5$



$2 \cdot 2 \cdot 3 \cdot 5 = 60$

The 60th customer will receive all three free items.

8. Make a list of the keywords or phrases you notice with LCM word problems.

"what is the first ___ to..." (answers will vary)

"least common multiple"

"smallest multiple"

SUMMARY (What I learned in this section)