SECTION 1.5 EQUIVALENCE OF EXPRESSIONS

Name: Key **Date:** ______ **Period:** ______

Vocabulary

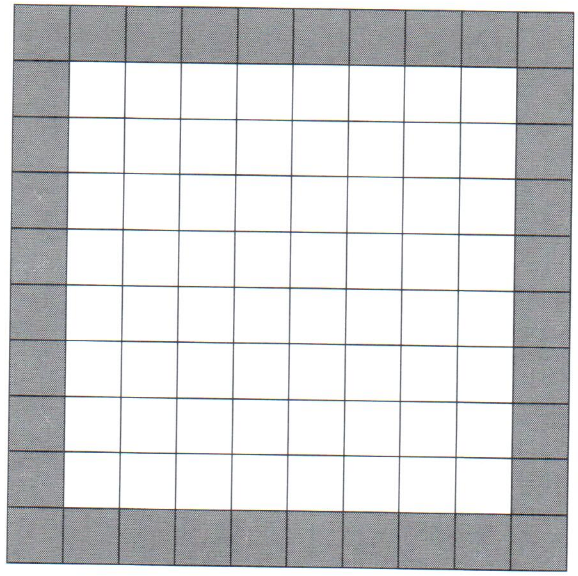
DEFINITION	EXAMPLE
Commutative Property of Addition For any two numbers x and y, $x+y=y+x$ (the order of addition does not matter)	$24+6=30=6+24$ $8+(-5)=(-5)+8$
Associative Property of Addition for any numbers x, y, and z: $(x+y)+z=x+(y+z)$ (grouping of addition does not matter)	$(3+4)+5=7+5=12$ $3+(4+5)=3+9=12$
Distributive Property for any numbers x, y, and z, $x(y+z)=xy+xz$	$3(2+4)=3\cdot6=18$ $3\cdot2+3\cdot4=6+12=18$
Multiplicative Inverse For any non-zero number n, $n\cdot\frac{1}{n}=1$ ($\frac{1}{n}$ is the multiplicative inverse, or reciprocal, of n.)	$5\cdot\frac{1}{5}=1$ ↑
Like Terms numbers or expressions sharing the same variable	3x and 2x 4 and 1 18z and 5z

EXPLORATION 1

Consider the square grid below, representing an 8×8 swimming pool with a shaded border of width 1. How many squares are shaded in? Answer this without talking, without counting one by one, and without writing.

Answer: 36

Work will vary (and will be mental)



$8+8+8+8+4$

$4 \cdot 8 + 4 \cdot 1$

$9 \cdot 4$

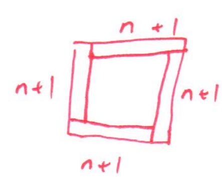
etc.

EXPLORATION 2

Now look at a $n \times n$ square swimming pool with a border of width one. Determine the number of squares in the border using two of the methods your classmates described in Exploration 1. For each of the methods write out in words what the method is doing. Write an algebraic expression that explains each method you used. Make sure to say what the variable in your expression represents.

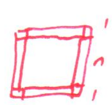
Answers will vary. S = shaded squares

There are 4 strips of $n+1$



$4(n+1) = S$

There are 4 edges of the pool with length n, and 4 corners.



$4n + 4 = S$

Each side could be counted up, along with the corners: $n+n+n+n+1+1+1+1 = S$

etc.

EXAMPLE 1

Write the following relationship mathematically:

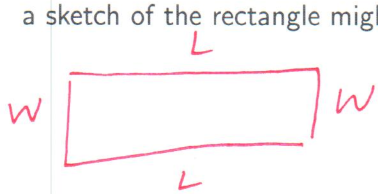
"Twice the amount of money that Jessica has in the bank now"

in two different ways. J = amount Jessica has in the bank now

$2J = J + J$

PROBLEM 2

A rectangle has length L and width W. What is the perimeter of the rectangle? Write as many expressions for the perimeter as you can and explain how you arrived at the expressions. Drawing a sketch of the rectangle might be helpful in the explanation.



$P = (L+L)+(W+W) = 2L+2W =$
$2(L+W) = (L+W)+(L+W) =$
$(W+W)+(L+L) = 2W+2L =$
$2(W+L) = (W+L)+(W+L)$ (etc.)

EXPLORATION 3

Determine which of the following number sentences are true. Try to justify your answer without calculating each side.

T 1. $5 + 7 = 7 + 5$ ✓ commutative prop. of add.

T 2. $56 + 89 = 89 + 56$ comutative prop of add.

T 3. $457 + 684 = 684 + 457$ 〃

T 4. $578943 + 674321 = 674321 + 578943$ 〃

F 5. $7 - 5 = 5 - 7$

F 6. $56 - 89 = 89 - 56$

F 7. $457 - 684 = 684 - 457$

F 8. $578943 - 674321 = 674321 - 578943$

T 9. $1 + 2 - 3 = 2 + 1 - 3$ associative prop. of add.

T 10. $4 + 5 - 6 = 5 - 6 + 4$ 〃

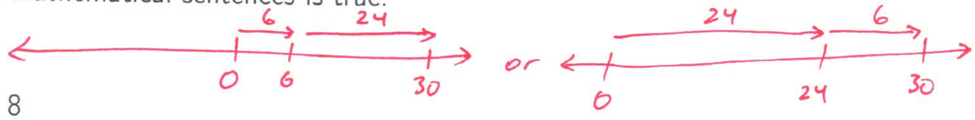
F 11. $11 - 14 + 21 = 11 - 21 + 14$

T 12. $11 + (-14) + 21 = 11 + 21 + (-14)$ associative prop. of add.

PROBLEM 3

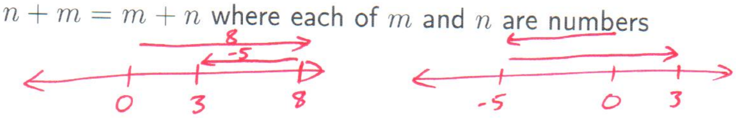
The number line is a good way to visualize properties of addition. Use a number line to illustrate that each of the following mathematical sentences is true.

1. $24 + 6 = 6 + 24$



2. $8 + (-5) = (-5) + 8$
3. $n + m = m + n$ where each of m and n are numbers

2.



3.

(examples will differ if m or n are negative)

PROBLEM 4

Is it possible to change the order with other operations and maintain equivalence? Check to see if this works with subtraction, multiplication, and division. For each operation, if it is commutative, make a rule. If not, give an example of how it fails to be commutative.

$3-1=2$ ✗ $\quad 3\cdot 4=12$ ✓ $\quad \frac{12}{4}=3$ ✗

$1-3=-2 \quad 4\cdot 3=12 \quad \frac{4}{12}=\frac{1}{3}$

For any two numbers x and y, $xy=yx$.

PROBLEM 5

1. Does the associative property work for subtraction? Is $(6 - 5) - 3 = 6 - (5 - 3)$? NO.

   $(6-5)-3=1-3=-2 \qquad 6-(5-3)=6-2=4$

2. Does multiplication have an associative property? Is $(8 \cdot 4) \cdot 2$ equal to $8 \cdot (4 \cdot 2)$? Is $(ab)c = a(bc)$? yes.

   $(8\cdot 4)\cdot 2=32\cdot 2=64 \qquad 8\cdot(4\cdot 2)=8(8)=64$

3. Does division have an associative property? Check to see if $(8 \div 4) \div 2$ and $8 \div (4 \div 2)$ are equivalent. No.

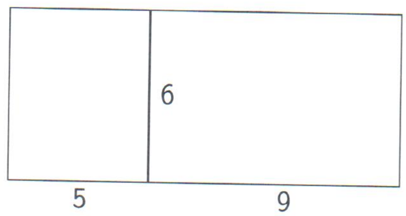
   $(8\div 4)\div 2=2\div 2=1 \qquad 8\div(4\div 2)=8\div 2=4$

4. Discuss why when carrying out any operation is it important to be particularly mindful of parentheses when subtracting and dividing.

   Because the order you subtract and divide changes the answers. They are not associative operations.

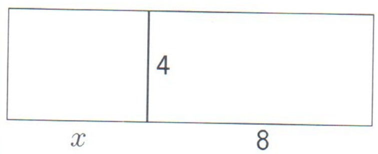
EXPLORATION 4

Compute the area of each of the large rectangles below in at least two ways:

1.


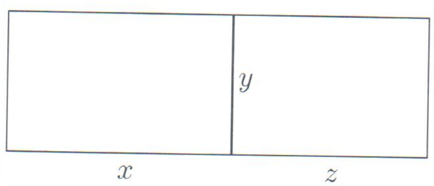
$5 \cdot 6 + 9 \cdot 6 = 30 + 54 = 84$

$6 \cdot (5+9) = 6 \cdot 14 = 84$

2.


$4 \cdot x + 4 \cdot 8 = 4x + 32$

$4(x+8) = 4x + 32$

3.


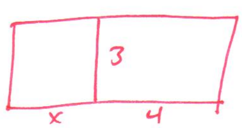
$xy + zy = xy + yz$

$y(x+z) = xy + yz$

PROBLEM 6

area of whole

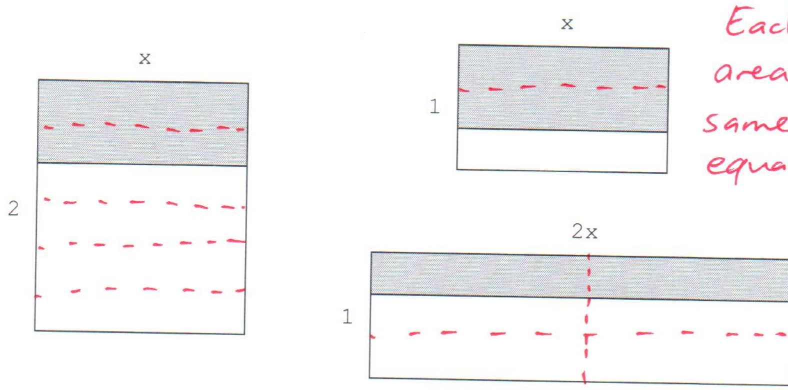
Use the area model to show that $3(x + 4) = 3x + 3 \cdot 4$.



part part

EXAMPLE 2

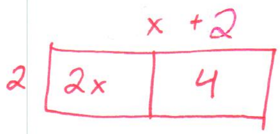
Use the three rectangles below to explain why the expressions $\frac{2x}{3}$, $\frac{2}{3}x$, and $\frac{1}{3}(2x)$ are equivalent.

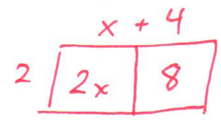


Each shaded area is the same number of equal parts.

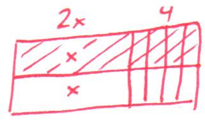
PROBLEM 7

Kate believes that $\frac{2x+4}{2} = x + 2$. Jamie says, "No. It should be $\frac{2x+4}{2} = x + 4$." Who is right? Draw an area model to support your answer.





Or:



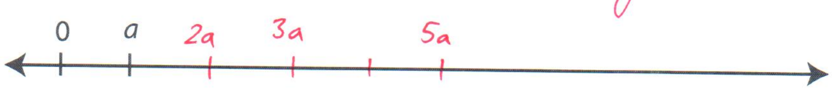
$\frac{1}{2} \cdot 4 = 2$

$x + 2$

EXPLORATION 5

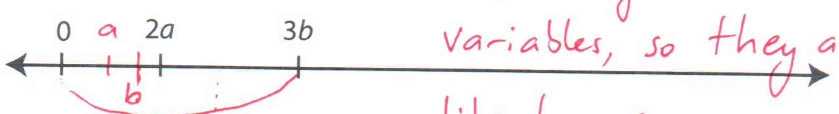
1. The number a is marked on the number line below. Locate $2a$, $3a$ and $5a$ on the number line. Explain why $2a + 3a = 5a$.

They are like terms.



2. The numbers $2a$ and $3b$ are marked on the number line. Locate a, b and $2a + 3b$ on the number line. In part 1, we "combined" $2a$ and $3a$ into $5a$. Do you think it is possible to write $2a + 3b$ as one term? Explain.

no, they have different variables, so they are not like terms.



EXAMPLE 5

Find an expression that is equivalent to $3(x + 4) - 2(2x - 3) + 8x - 1$ by:

1. Using the distributive property to remove the need for parentheses,
2. Combining like terms.

$3(x+4) - 2(2x-3) + 8x - 1$

$3x + 12 + (-2)(2x-3) + 8x - 1$

$3x + 12 + (-4x) - (-6) + 8x - 1$

$3x + 12 - 4x + 6 + 8x - 1$

$\rightarrow 7x + 17$

PROBLEM 8

Match each expression on the left to an equivalent expression on the right. Explain.

1. $4(2x-3)+3x-2$ b $8x-12+3x-2$	a. $-(x+18)$ $= -x-18$
2. $2(3-x)+3(x+5)-1$ d $6-2x+3x+15-1$	b. $11x-14$
3. $5(2x-3)-2(3-2x)$ e $10x-15-6+4x$	c. $8(x+3)$ $= 8x+24$
4. $7(x+3)+3(x+3)-2x-6$ c $7x+21+3x+9-2x-6$	d. $x+20$
5. $4x-5(x-2)+8$ a $4x-5x+10+8$	e. $14x-21$

Number Sense, Mental Math and Equivalent Expressions

EXPLORATION 6

1. Compute each of the following.
   a. $54 \div 10$ 5.4
   b. $54 \div 100$ .54
   c. $54 \div 1000$ .054

2. A fourth grader is learning how to divide and is given the problems above. How would you explain the "easy way" to find the answer? Answers may vary:
   Shift the decimal place

3. Compute each of the following. What do you notice?
   a. $100 \div 50$ 2
   b. $10 \div 50$ .2
   c. $27 \div 50$ $= 54 \div 100 = .54$
   d. $132 \div 50$ $= 264 \div 100 = 2.64$

4. Use the properties in this chapter to show that $x \div 50 = 2x \div 100$.
   $\frac{x}{50} = \frac{2}{2} \cdot \frac{x}{50} = \frac{2x}{100}$

5. Discuss with your neighbor how $x \div 50 = 2x \div 100$ can be used to compute $1234 \div 50$. Do you think this is easier than using long division? $\frac{1234}{50} = \frac{2468}{100}$ so $1234 \div 50 = 24.68$

6. Naveen is traveling in India to visit his grandparents. The exchange rate is 50.5 Indian Rupees to the dollar. He sees a shirt he wants to buy. The price is 332 rupees. Naveen wants to know if this is good price. Use the trick you explored here to estimate how much the shirt costs in dollars.

50.5 is about 50, so

$$\frac{2 \text{ dollars}}{100 \text{ rupees}} = \frac{1 \text{ dollar}}{50 \text{ rupees}} = \frac{x \text{ dollars}}{332 \text{ rupees}}$$

$$\frac{2 \text{ dollars}}{100 \text{ rupees}} \approx \frac{x \text{ dollars}}{300 \text{ rupees}} \approx \frac{3 \cdot 2 \text{ dollars}}{3 \cdot 100 \text{ rupees}}$$

About \$6

SUMMARY (What I learned today)