SECTION 1.6 Equivalent Equations

Name: Key

Vocabulary

DEFINITION	EXAMPLE
Solution of an Equation A value of the variable that makes both sides of the equation equal.	$x = 3$ is a solution of $x+2=5$
Empty Solution Set when no value of a variable makes the equation true	$x+3=x-1$ has an empty solution set

EXPLORATION 1

Answers will vary

1. Write two different expressions which are equivalent. Explain why they are equivalent.

$4(2x+3)=8x+12$ when you multiply & simplify, they are both $8x+12$.

2. Write two different equations which are equivalent. Explain why they are equivalent.

$2x=4$ and $x=2$ both equations have the same solution

EXAMPLE 1

Solve the following equation, explaining each step and naming the property you use:

$$3x + 1 = x - 7.$$

$3x+1-x=x-7-x$ subtraction property of equality

$2x+1=-7$ combining like terms

$2x+1-1=-7-1$ subtraction property of equality

$2x=-8$ combining like terms

$\frac{2x}{2}=\frac{-8}{2}$ division property of equality

$x=-4$ simplify

PROBLEM 1

Solve the following equations:

1. $2(2x+2)+3(x+2)=x-2.$

$4x+4+3x+6=x-2$
$7x+10=x-2$
$7x+10=x-2$
$-10 \quad -10$
$7x=x-2-10$
$7x-x=x-12-x$
$6x=-12$
$\frac{6x}{6}=\frac{-12}{6}$
$x=-2$

2. $3(x+5)-(x-3)=2(x-3).$

$3x+15-x-(-3)=2x-6$
$2x+18=2x-6$
$2x+18-2x=2x-6-2x$
$18=-6$
$\therefore$ empty solution set

3. $-2(x-3)+4x=5x-7.$

$-2x+6+4x=5x-7$
$2x+6=5x-7$
$2x+6-2x=5x-7-2x$
$6=3x-7$
$6+7=3x-7+7$
$13=3x$
$\frac{13}{3}=\frac{3x}{3}$
$\frac{13}{3}=x$

EXPLORATION 2

1. Consider the equation $5(2x-3)-4x=6x-15.$

a. Try to solve the equation. What happens?

$5(2x-3)-4x=6x-15$
$10x-15-4x=6x-15$
$6x-15=6x-15$

b. Substitute $x=1$ on both sides of the equation. What happens? Substitute $x=0$. Now what happens?

$6(1)-15=6(1)-15$
$6-15=6-15$
$-9=-9$ ✓

$6(0)-15=6(0)-15$
$0-15=0-15$
$-15=-15$ ✓

c. Use the distributive property and combining like terms to show that $5(2x-3)-4x$ is equivalent to $6x-15$. What does this say about which values of x make the equation true?

$5(2x-3)-4x$
$=10x-15-4x$ by distributive property
$=6x-15$ by combining like terms

the two sides of the equation are the same so the solution set is all numbers

2. Consider the equation $3(x-2)+2x=5x+4$.

a. Try to solve the equation. What happens?

We end up with $-6=4$, which is never true, so the solution set is empty.

$3(x-2)+2x=5x+4$
$3x-6+2x=5x+4$
$5x-6=5x+4$
$5x-6-5x=5x+4-5x$
$-6=4$:(

b. Use the distributive property and combining like terms to show that $3(x-2)+2x$ is equivalent to $5x-6$.

$3(x-2)+2x$
$=3x-6+2x$
$=5x-6$

c. Is there any value of x so that $5x-6$ is the same as $5x+4$? Explain. What does this say about which values of x make the equation $3(x-2)+2x=5x+4$ true?

No, because $5x=5x$, so <u>if</u> $5x-6=5x+4$, then $5x-6-5x=5x+4-5x$

so $-6=4$

which is not true.

There are no values of x that make $3(x-2)+2x=5x+4$ true, so the solution set is empty.

EXAMPLE 3

Suppose n is a number so that if you triple it and subtract 4 you obtain the same number as if you decrease it by 3 and then double the result. What is n?

$3n - 4 = 2(n-3)$

$3n - 4 = 2n - 6$

$3n - 4 - 2n = 2n - 6 - 2n$

$n - 4 = -6$

$n - 4 + 4 = -6 + 4$

$\boxed{n = -2}$

PROBLEM 2

Montserrat has two job offers to deliver fliers around the neighborhood. The first offers to pay her \$50 per week plus $10\frac{1}{2}$ cents per flier. The second will pay only \$30 per week, but will give 20 cents per flier.

1. Set up an equation to find x the number fliers she must deliver so that the two offers pay the same per week.
2. Solve for x. Which job would you take and why?

$50 + .105x = 30 + .20x$

$50 + .105x - .105x = 30 + .20x - .105x$

$50 = 30 + .095x$

$50 - 30 = 30 + .095x - 30$

$20 = .095x$

$\frac{20}{.095} = \frac{.095x}{.095}$

$210.5 \approx x$

$211 \approx x$

The second job will pay more after 211 fliers have been delivered, so it depends on how many fliers she delivers.

SUMMARY (What I learned today)