SECTION 1.7 FORMULAS AND LITERAL EQUATIONS

Name: Key   **Date:** ________   **Period:** ________

Vocabulary

DEFINITION	EXAMPLE
Formula "literal equation" or equations with 2 or more variables or letters	$P = 2L + 2W$ $C = 2\pi r$ or $C = \pi d$

EXPLORATION 1

What formulas do you know? Which have you worked with in other grades? Which formulas have you used in other classes besides math such as science? Remember to define what each variable in a formula means.

Area of a rectangle = A
Length = L
Width = W

$A = L \cdot W$

Area of a triangle = A
b = base of triangle
h = height of triangle

$A = \frac{1}{2} \cdot b \cdot h$

EXPLORATION 2

Use $P = 2L + 2W$ and find two equations equivalent to it, one that has W by itself on one side of the equal sign and the other that has L by itself on one side of the equal sign. In other words, isolate W, solve the equation for W, or express W in terms of the other variables. Also do this for L. Explain each of your steps.

$P = 2L + 2W$
$P - 2W = 2L + 2W - 2W$
$P - 2W = 2L$
$\frac{P}{2} - \frac{2W}{2} = \frac{2L}{2}$
$\frac{P}{2} - W = L$

OR

$P = 2L + 2W$
$P - 2L = 2L + 2W - 2L$
$P - 2L = 2W$
$\frac{P}{2} - \frac{2L}{2} = \frac{2W}{2}$
$\frac{P}{2} - L = W$

OR

$P = 2L + 2W$
$P = 2(L + W)$

EXAMPLE 1

The area of a rectangle is 54 square meters. Its length is 3 meters. Determine its width.

$A = 54$ $L = 3$

$A = L \cdot W$

$54 = 3 \cdot W$

$\frac{54}{3} = \frac{3W}{3}$

$18 = W$

The width is 18 meters

PROBLEM 2

The area of Cody's triangle is 20 square inches. The length of its base is 8 inches. The area of Althea's triangle is 15 square inches. The length of its base is 5 inches. What is the height of each of these triangles?

A_C = area of Cody's triangle
b_C = base of Cody's triangle
h_C = height of Cody's triangle

$A_C = \frac{1}{2} b_C \cdot h_C$

$20 = \frac{1}{2} \cdot 8 \cdot h_C$

$20 = 4 \cdot h_C$

$\frac{20}{4} = \frac{4h_C}{4}$

$5 = h_C$

A_A = area of Althea's triangle
b_A = base of Althea's triangle
h_A = height of Althea's triangle

$A_A = \frac{1}{2} b_A \cdot h_A$

$15 = \frac{1}{2} \cdot 5 \cdot h_A$

$15 = 2.5 h_A$

$\frac{15}{2.5} = \frac{2.5 h_A}{2.5}$

$6 = h_A$

Althea's triangle's height is 6 in.
Cody's triangle's height is 5 in.

EXPLORATION 3

A box has a square base of length x. The height of the box is 3 times the length of the base. Write an expression for each of the following:

1. Volume of the box. $= V$ length $= l$ width $= w$ height $= h$

   $V = lwh$ $V = (x)(x)(3x) = 3x^3$ $\boxed{V = 3x^3}$

2. Surface area of the box. $= SA$

   $SA = 2(lw) + 2(wh) + 2(lh)$ or $SA = 2(lw + wh + lh)$

Now let's explore the data you collected in Exercise 6 from Section 1.6.

EXPLORATION 4

Recall the formula for the *circumference* C of a circle in terms of its *radius* r, $C = 2\pi r$ and in terms of its *diameter* d, $C = \pi d$.

1. What is the relationship between d and r? Write an equation to represent this relationship.

   $d = 2r$ The diameter is double the radius

2. Solve the equation $C = \pi d$ for π in terms of C and d.

   $\frac{C}{d} = \frac{\pi d}{d}$ $\boxed{\pi = \frac{C}{d}}$

3. For Exercise 6 from Section 1.6 you measured the circumference and diameter of the base of some cylinders you found in your house. Discuss with your neighbors why you chose the objects you did, how your measured the circumference and any difficulties that arose.

Answers will vary.

4. As a group record the measurements of the objects from around your house. Using a calculator compute $\frac{C}{d}$ for each object. What should this equal?

Name of Object	d (mm)	C (mm)	$\frac{C}{d}$
Answers will vary.			$\approx \pi$
			$\approx \pi$
			$\approx \pi$
			$\approx \pi$

5. Use the results from your table to estimate the value of π.

Answers will be approximately 3.14

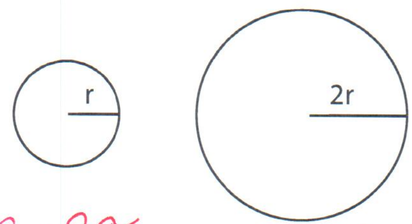
6. Press the π key on your calculator. How does this number compare to your estimate above?

It will have more digits and be closer to π.

EXAMPLE 2

If we examine a second circle with radius twice the radius of the first, what is the relationship between the circumference of the second circle to the circumference of the first?

$C = 2\pi r$



$C_1 = 2\pi r$ $\qquad C_2 = 2\pi(2r)$

$C_1 = 2\pi r$ $\qquad C_2 = 4\pi r$

The circumference of the second circle is twice the circumference of the first circle

PROBLEM 3

What happens to the circumference of a circle if the radius of a circle is tripled? What happens to the circumference of a circle if the radius of a circle is quadrupled? Do you see a pattern?

$C_3 = 2\pi(3r)$ $\qquad C_4 = 2\pi(4r)$

$C_3 = 6\pi r$ $\qquad C_4 = 8\pi r$

(triples) (quadruples)

yes, radius increased by a factor of n increases C by a factor of n.

PROBLEM 4

Consider a box with length L, width W and height h. The formula for the volume is $V = LWH$.

1. If the volume is 20 cubic centimeters, what is the formula for the width W?

$V = LWH$

$20 = LWH$ $\qquad \frac{20}{LH} = \frac{LWH}{LH}$ $\qquad \frac{20}{LH} = W$

2. If the surface area is 60 square centimeters what is the formula for the height H?

$60 = 2(LW + WH + LH)$

$\frac{60}{2} = LW + WH + LH$

$30 - LW = LW + WH + LH - LW$

$30 - LW = WH + LH$

$30 - \cancel{LW} = H(W+L)$

$\frac{30 - LW}{(W+L)} = \frac{H(W+L)}{(W+L)}$

$\frac{30}{(W+L)} - \frac{LW}{(W+L)} = H$

PROBLEM 5

For each of the following situations:

- determine the formula to use
- specify what each variable means
- compute the area or volume

1. A model of a square pyramid has base edges of 10 inches and height of 12 inches. The slant height is 13 inches. What is its lateral area? Pyramid lateral Surface Area

   $S = \frac{1}{2} P l$ P = perimeter of base, l = slant height

   S = lateral surface area

   $S = \frac{1}{2}(10 \cdot 4) \cdot 13$

   $S = 260$ sq. in.

2. What is the volume of a sphere with a 6cm radius?

   $V = \frac{4}{3}\pi r^3$ $V = \frac{4}{3}\pi 6^3$ $V = 288$ in^2

   V = volume, r = radius

3. What is the volume of a cone with a height of 15 inches and a radius of 4 inches for its circular base?

   $V = \frac{1}{3} B h$ V = volume, B = area of cone's base, h = height

   $B = \pi r^2$

   $B = \pi (4^2)$

   $B = 16\pi$

   $V = \frac{1}{3}(16\pi)(15)$

   $V = 80\pi$ in^3

Formulas from Geometry

CIRCUMFERENCE		
Circle	$C = 2\pi r$	$C = \pi d$
AREA		
Triangle		$A = \frac{1}{2}bh$
Rectangle or Parallelogram		$A = bh$
Trapezoid		$A = \frac{1}{2}(b_1 + b_2)h$
Circle		$A = \pi r^2$
SURFACE AREA		
	Lateral	Total
Prism	$S = Ph$	$S = Ph + 2B$
Pyramid	$S = \frac{1}{2}Pl$	$S = \frac{1}{2}Pl + B$
Cylinder	$S = 2\pi rh$	$S = 2\pi rh + 2\pi r^2$
VOLUME		
Prism or cylinder		$V = Bh$
Pyramid or cone		$V = \frac{1}{3}Bh$
Sphere		$V = \frac{4}{3}\pi r^3$

SUMMARY (What I learned today)