SECTION 2.3 PATTERNS AND SEQUENCES

Name: Key **Date:** ________ **Period:** ________

Vocabulary

DEFINITION	EXAMPLE
Sequence range of a function whose domain is the natural numbers, list of numbers (each number in the sequence is a "term")	n | 1 2 3 4 5 A(n) | 4 7 10 13 16
Arithmetic Sequence a sequence $a_1, a_2, a_3, \ldots$ such that for each natural number n, there is a c where $a_{n+1} = a_n + c$ or $a_{n+1} - a_n = c$	4 4 4 4 ^ ^ ^ ^ -3, 1, 5, 9, 13, ...
Recursive Formula a relationship in terms of consecutive terms such as a_n and a_{n+1}	for 4, 7, 10, 13, 16, ... $a_{n+1} = a_n + 3$ is the recursive form of $A(n) = 3n + 1$

EXAMPLE 1

Let the function A be defined by $A(n) = 3n + 1$ for all natural numbers n.

1. Complete the table below:

n	1	2	3	4	5
$A(n)$	4	7	10	13	16

2. Write the outputs of the function as a list.

4, 7, 10, 13, 16, ...

3. When a sequence is written as a list, each member of the sequence is called a *term* of the sequence. Compare consecutive terms (terms that are next to one another) in the sequence. What relationship do you notice? Use this relationship to find the next three terms in the sequence.

each term is 3 more than the previous term

4, 7, 10, 13, 16, 19, 22, 25

EXPLORATION 1

Find a formula for the n-th term of the sequence:

$$1, 3, 5, 7, 9, 11, \ldots$$

1. Compare consecutive terms in the sequence. Describe the relationship between them. Use this relationship to find the next two terms in the sequence.

$$\overset{}{1}, \overset{+2}{3}, \overset{+2}{5}, \overset{+2}{7}, \overset{+2}{9}, \overset{+2}{11}, \underline{13}, \underline{15}, \ldots$$

each term is 2 more than the previous term

2. Make a table like the one used in Example 1.

n	1	2	3	4	5	6	7	8
$A(n)$	1	3	5	7	9	11	13	15

3. Find the formula for the n-th term of the sequence.

$a_1 = A(1) = 1$

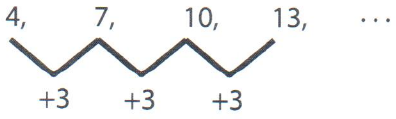
$a_2 = A(2) = 1 + 2$

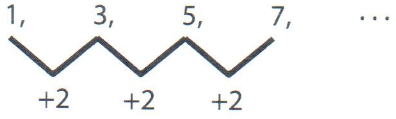
$a_3 = A(3) = 1 + 2 + 2$

$$A(n) = 1 + (2n - 2) = 1 + 2n - 2 = 2n - 1$$

$$A(n) = 2n - 1$$

The two sequences considered so far had similar recursive formulas. The differences between consecutive terms were constant.





Sequences with this type of recursive formula have a special name: Arithmetic Sequences.

PROBLEM 1

Consider the sequences:

- 2, 4, 6, 8, . . . +2 each time 1) yes

2)

n	1	2	3	4
A(n)	2	4	6	8

- −3, 1, 5, 9, 13, . . . +4 each time 1) yes 2)

n	1	2	3	4	5
An	-3	1	5	9	13

- $\overset{3}{\wedge}\ \overset{5}{\wedge}\ \overset{7}{\wedge}$ 2, 5, 10, 17, . . . 1) no. 2)

n	1	2	3	4
an	2	5	10	17

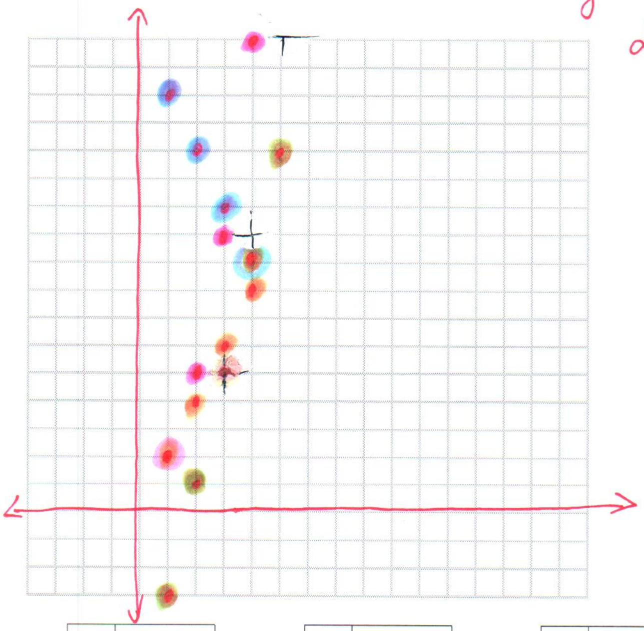
- 15, 13, 11, 9, . . . −2 each time 1) yes (-2 can be thought of as +(-2) instead)

2)

n	1	2	3	4
an	15	13	11	9

For each sequence,

1. Determine if its arithmetic or not.
2. Make a table using the natural numbers as inputs and the sequence as outputs.
3. Graph the points of the table on a coordinate plane. Compare the graphs of the arithmetic sequences, how are they similar? They are in straight lines, but we do not connect them



n	a_n
1	2
2	4
3	6
4	8

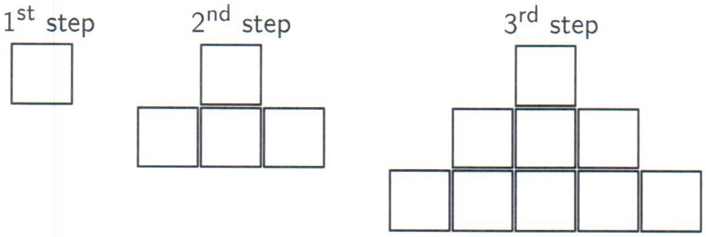
n	a_n
1	-3
2	1
3	5
4	9
5	13

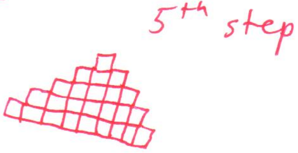
n	a_n
1	2
2	5
3	10
4	17

n	a_n
1	15
2	13
3	11
4	9

EXPLORATION 2

Suppose that you have a large supply of blocks. Make a sequence of stacks of blocks in the following way. First you place one block down. In step 2, place three blocks in a row under the first block, centered horizontally. In step 3, place five blocks in a row under the stack in step 2 to form a new stack.





1. Draw the next two stacks in steps 4 and 5 by adding a new row of blocks with two more blocks than the row above it.
2. If we continued this process, make a list of what changes. (Try to find five different things that change.) In groups, share your lists and make a group list. Answers will vary
3. We are interested in changes that can be described by a numerical value. For each quantity that changes, we want to define a function that gives us this numerical value as its output. The input for each of these outputs will be the step number. For example, we let $h(n)$ be the height of the stack at step n. What is $h(1)$ and $h(2)$? For another example, we let $B(n)$ denote the number of blocks in the n^{th} step.
4. In each group, pick 3 things that change from the class list. Make a table for each of these functions. Try to find a rule for at least one of these functions.

2) number of blocks = $B(n)$
number of stairs
number of edges total
number of interior edges
height = $h(n)$
width = $w(n)$
etc.

3) $h(1) = 1$
$h(2) = 2$

4)

n	$B(n)$
1	1
2	4
3	9
4	16
5	25

n	$h(n)$
1	1
2	2
3	3
4	4
5	5

n	$w(n)$
1	1
2	3
3	5
4	7
5	9

$B(n) = x^2$ $h(n) = n$ $w(n) = 2n - 1$

PROBLEM 2

For any function f whose domain is the set of all numbers or all positive numbers, you could restrict the domain to the natural numbers and produce the sequence

$$f(1), f(2), f(3), f(4), f(5), f(6), \ldots$$

For each of the following functions restrict the domain to the natural numbers and:

- write out the first 4 members of the sequence
- determine if the sequence is arithmetic or not
- use the pattern to write the next two members of each sequence without computing with the rule

1. $f(x) = x^2 + 1$

   3 5 7 9 11

   2, 5, 10, 17, 26, 37, ...

   not arithmetic

2. $f(x) = 2x + 3$

   2 2 2 2 2

   5, 7, 9, 11, 13, 15, ...

   arithmetic

3. $f(x) = 3x + 2$

   3 3 3 3 3

   5, 8, 11, 14, 17, 20, ...

   arithmetic

4. $f(x) = \frac{1}{x}$

   $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots$

   not arithmetic

5. $f(x) = \frac{1}{2x-1}$

   $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \ldots$

   not arithmetic

6. $f(x) = 2^x$

   2, 4, 8, 16, 32, 64, ...

   not arithmetic

SUMMARY (What I learned today)