SECTION 3.3 Slope and Proportions

Name: Key **Date:** _____ **Period:** _____

Vocabulary

DEFINITION	EXAMPLE
Proportional Relationship The ratio is always the same. Also called direct variation. The graph is a line through the origin.	$\frac{y}{x} = m$ or $y = mx$ m is the constant ratio
Constant of Proportionality The constant ratio between two variables (slope of the line of the graph of a proportional relationship.)	$y = 3x$ ↑

EXPLORATION 1

A restaurant makes and sells a famous dish that contains rice and beans. The ratio of rice to beans in its secret recipe is 1 : 2, and the ratio of beans to rice is 2 : 1.

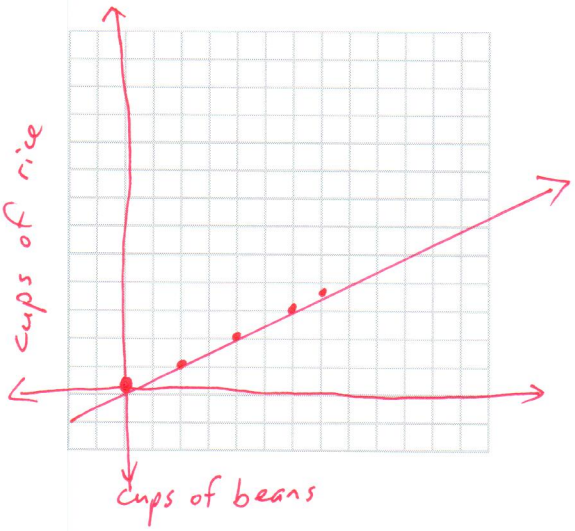
Beans(cups) x	Rice (cups) y	$\frac{y}{x}$
2	1	$\frac{1}{2}$
4	2	$\frac{2}{4} = \frac{1}{2}$
6	3	$\frac{3}{6} = \frac{1}{2}$
7	3.5	$\frac{3.5}{7} = \frac{1}{2}$

1. Fill in the table of possible amounts of rice and beans that the chef uses to make the dish.

2. For each (x, y) pair compute the ratio of $\frac{y}{x}$. What do you notice about this ratio?

It is the same for each pair.

3. Use the numbers in the table as coordinates of points. Make a graph using the data points. For each point, the number of cups of beans is the x-coordinate and the number of cups of rice is the y-coordinate. Describe the graph.



A line passing through the origin with slope $m = \frac{1}{2}$

4. Write the equation of the line through the points.

$y = \frac{1}{2}x$

5. What is relationship between slope of the line and the ratio from part 2.

They are both $\frac{1}{2}$.

6. Use the equation of the line to find:
   a. If we want to use 13 cups of beans, how many cups of rice do we need?

$y = \frac{1}{2}(13)$ $y = 6.5$ 6.5 cups of rice

   b. If we want to us 10 cups of rice, how many cups of beans do we need?

$10 = \frac{1}{2}x$ $10 \cdot 2 = \frac{1}{2} \cdot 2x$ $20 = x$

20 cups of beans

   c. If we want to use 15 cups of rice and beans altogether, how many cups of beans do we need?

$x + y = 15$

so $15 - x = y$ so $15 - x = \frac{1}{2}x$

$2(15 - x) = \frac{1}{2} \cdot 2 \cdot x$

$30 - 2x = x$

$30 - 2x + 2x = x + 2x$

$30 = 3x$

$10 = x$

$y = 15 - (10) = 5$

7. Explain how you could use the graph to answer 6a and 6b.

6a: look where $x = 13$, see that y is 6.5.

6b: look where $y = 10$, see that x is 20.

PROBLEM 1

Consider a basic recipe for vinaigrette which is 3 parts oil to 2 parts vinegar.

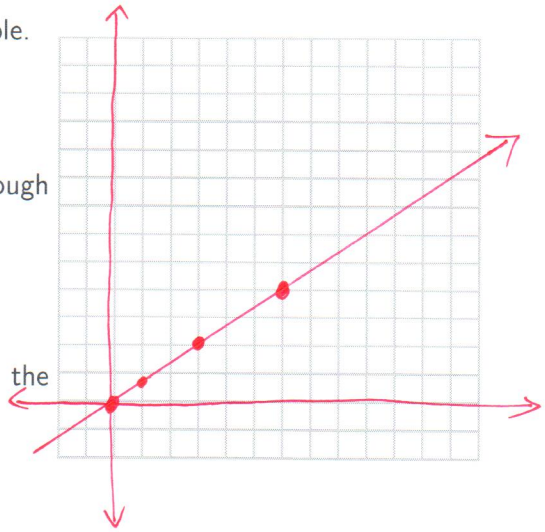
1. Make a table of amounts of oil (x) and vinegar (y) in tablespoons.

Oil x	Vinegar y	$\frac{y}{x}$
3	2	$\frac{2}{3}$
6	4	$\frac{6}{4} = \frac{2}{3}$
0	0	can't divide by 0.
1	$\frac{2}{3}$	$\frac{\frac{2}{3}}{1} = \frac{2}{3}$

2. Compute the ratio of $\frac{y}{x}$ for each pair in your table.

always $\frac{2}{3}$

3. Graph the ordered pairs in your table.



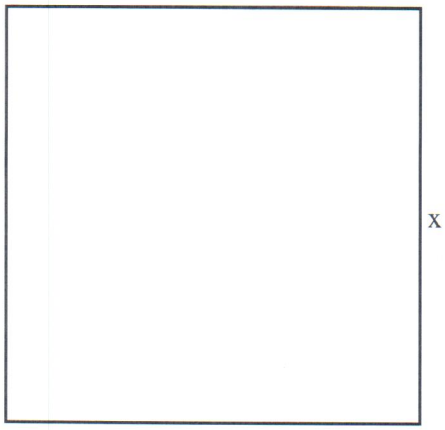
4. Write the equation of the line through the points.

$y = \frac{2}{3}x$

5. What is the relationship between the slope and ratio from part 2?

They are both $\frac{2}{3}$.

EXAMPLE 1



Let y = perimeter of the square.

1. Write an equation for the perimeter of the square in terms of x.

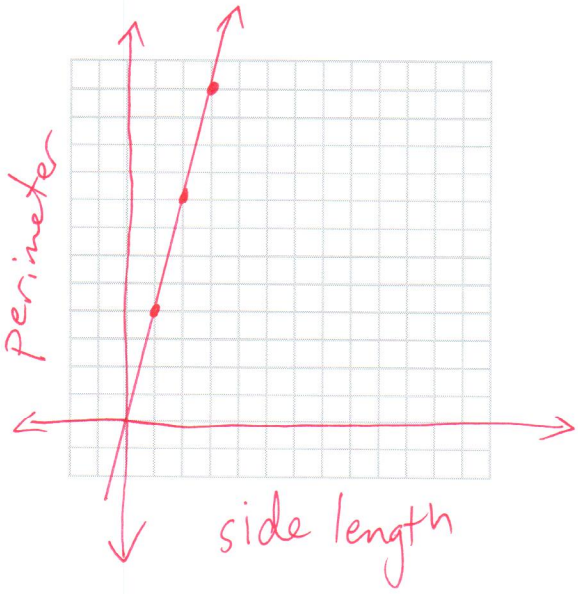
   $y = x + x + x + x$
   or $y = 4x$

2. For a square, is the relationship between the perimeter and the length of a side proportional?

   yes. $\frac{y}{x}$ is always 4.

3. Make a graph of the perimeter versus the length of a side. Does the graph cross at the origin? Explain why this makes sense.

   yes, $y = 4(0) = 0$ so when $x = 0$, $y = 0$.

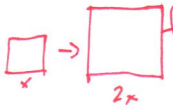


4. If we double the length of the sides, how does this affect the perimeter? What if we triple the length of sides? Draw a sketch to justify your answer.

   $y = f(x) = 4x$

   $f(2x) = 4(2x)$

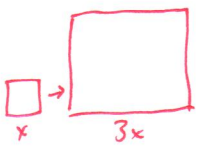
   $f(2x) = 8x$, which is double the original perimeter.



   $f(3x) = 4(3x)$

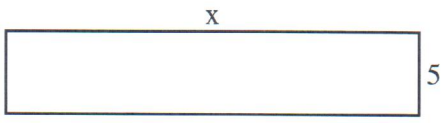
   $f(3x) = 12(x)$

   which is triple the original perimeter.



EXPLORATION 2

Let y be the perimeter of the rectangle shown below.

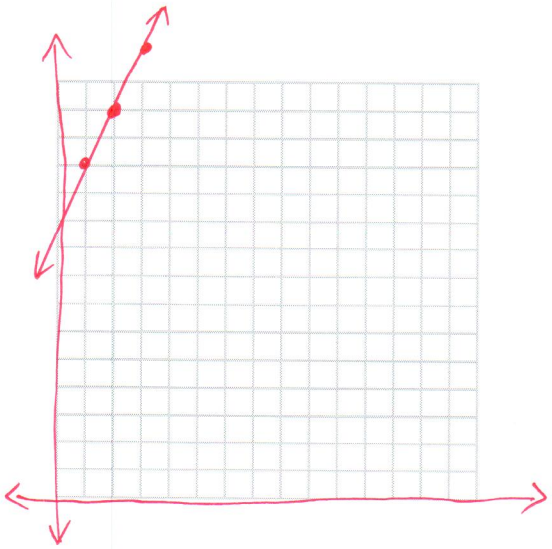


length x	perimeter y	$\frac{y}{x}$
2	14	$\frac{14}{2} = \frac{7}{2}$
3	16	$\frac{16}{3}$
4	18	$\frac{18}{4} = \frac{9}{2}$
8	26	$\frac{26}{8} = \frac{13}{4}$
5	20	$\frac{20}{5} = 4$
1	12	

1. Make a table of possible values of x and the corresponding perimeter y.

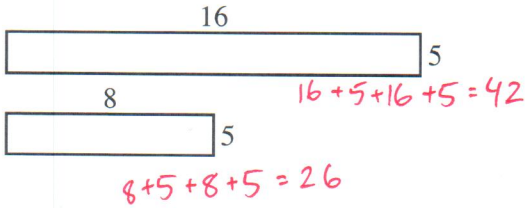
2. Is the perimeter proportional to x? For each pair (x, y) compute the ratio $\frac{y}{x}$? How does this compare to what happened in Example 1?

No, $\frac{y}{x}$ is not a constant. It is different for different pairs



3. Plot the points on a grid. Notice that the points fall along a line. Draw the line. Does the line pass through the origin? Explain why this makes sense.

No. It is linear but not a proportional relationship.



16 + 5 + 16 + 5 = 42

8 + 5 + 8 + 5 = 26

4. What happens to the perimeter if we double the length but leave the width the same? The rectangles shown have width 5. One has length 8 and the other length 16. Determine the perimeter for each rectangle. Did the perimeter double?

No, it did not.

Let's summarize some key results about proportional relationships:

1. The graph of the relationship is a line that passes through the origin.
2. The slope of the line is equal to the constant ratio between the two variables. Sometimes this is called the *constant of proportionality*.
3. If one variable is multiplied by a scale factor, the other variable is multiplied by the same scale factor.

EXPLORATION 3

Think of different contexts from your previous math classes or from your experience outside of school. Identify 5 relationships that are proportional.

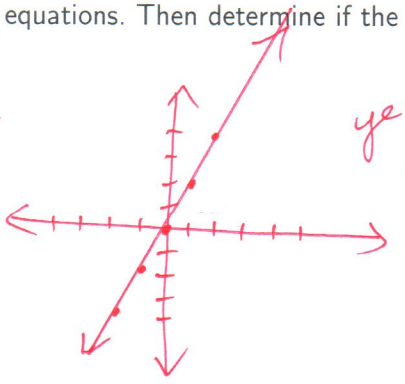
Answers will vary.
Scaling in geometry, miles per gallon, doubling (or changing) a recipe, sales tax, cost per pound, etc.

PROBLEM 2

Graph each of the following equations. Then determine if the relationship is proportional or not. Explain.

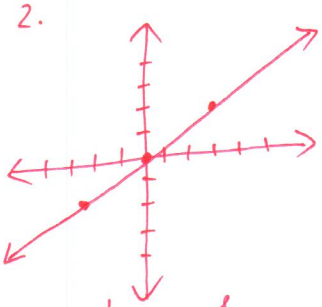
1. $y = 2x$
2. $y = \frac{2}{3}x$
3. $y = x^2$
4. $y = 2x + 3$

1.



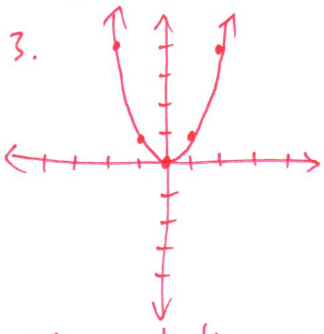
yes, linear & passes through origin.

2.



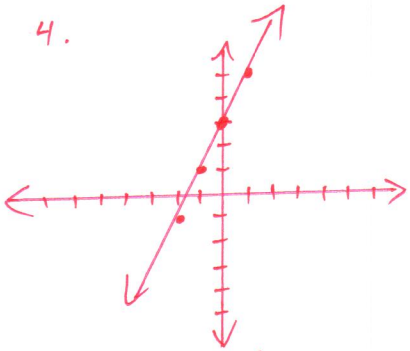
yes, linear & passes through the origin.

3.



No, not linear.

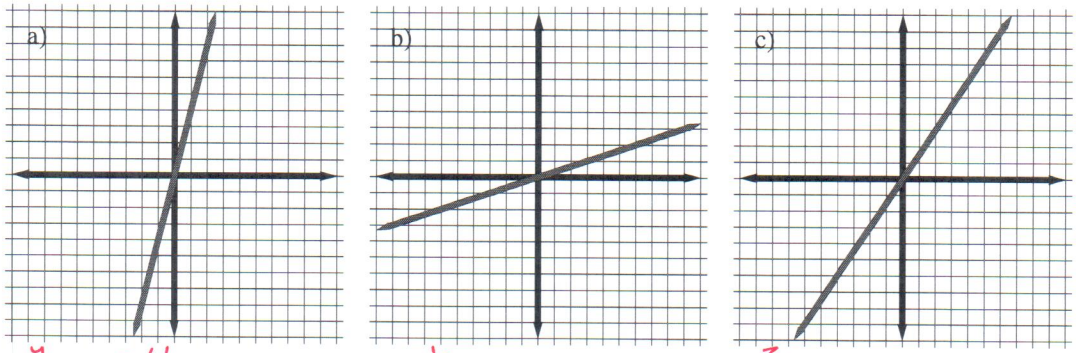
4.



No, does not pass through the origin.

PROBLEM 3

Explain why all the graphs below represent proportional relationships. For each graph, determine the slope and write the equation of the line.



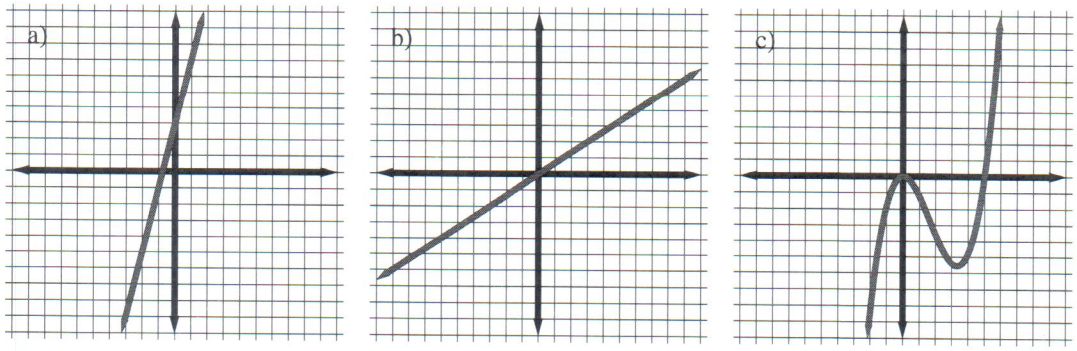
$\frac{y}{x} = m = 4$ $y = 4x$

$\frac{y}{x} = m = \frac{1}{3}$ $y = \frac{1}{3}x$

$\frac{y}{x} = m = \frac{3}{2}$ $y = \frac{3}{2}x$

PROBLEM 4

For each of the graphs below determine if the graph represents a proportional relationship or not.



No. Does not pass through the origin.

Yes.

No. Not linear.

SUMMARY (What I learned today)