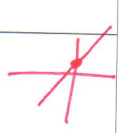
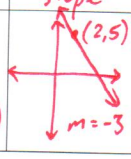
SECTION 3.4 SLOPES AND INTERCEPTS

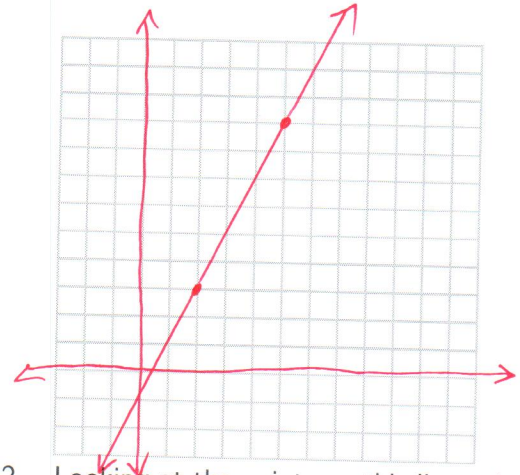
Name: Key **Date:** _____ **Period:** _____

Vocabulary

DEFINITION	EXAMPLE
Slope-intercept form $y = mx + b$ where m is the slope of the line & b is the y-intercept of the line	(see below)
y-intercept The value of the y-coordinate of the point where the line intersects the y-axis	$y = 3x + 2$ slope, y-intercept 
Point-slope Form $(y - y_1) = m(x - x_1)$ where m is the slope of the line & the line passes through (x_1, y_1)	 $(y-5) = -3(x-2)$

EXPLORATION 1

1. Plot the points $(2, 3)$ and $(5, 9)$. Draw a straight line passing through both of these points. Consider the points on this line, complete the table below.



First coordinate	Second coordinate
−2	-5
−1	-3
0	-1
1	1
2	3
3	5
4	7
5	9
6	11
7	13
8	15

2. Looking at the points on this line, what pattern do you see? For each ordered pair (x, y) in this table, describe in words the relationship between x and y. Write an equation that can be used to test if a point is on the line.

y-coordinates are the odd numbers, shifted

y-coordinates are doubled x minus 1

$y = 2x - 1$

EXPLORATION 2

Use the point tester " a point (x, y) is on the line if $y = 2x - 1$" to determine which of the following points are on the line from Exploration 1.

a. $(-1, -1)$

$(-1) = 2(-1) - 1$

$-1 = -2 - 1$

$-1 = -3$ X

b. $(-3, -6)$

$(-6) = 2(-3) - 1$

$-6 = -6 - 1$

$-6 = -7$

X

c. $(2) = 2(0) - 1$

$2 = -1$

X

d. $(1) = 2(4) - 1$

$1 = 8 - 1$

$1 = 7$

X

e. $(\frac{1}{2}) = 2(\frac{1}{2}) - 1$

$\frac{1}{2} = 1 - 1$

$\frac{1}{2} = 0$ X

f. $(-1.2) = 2(-1) - 1$

$-1.2 = -2 - 1$

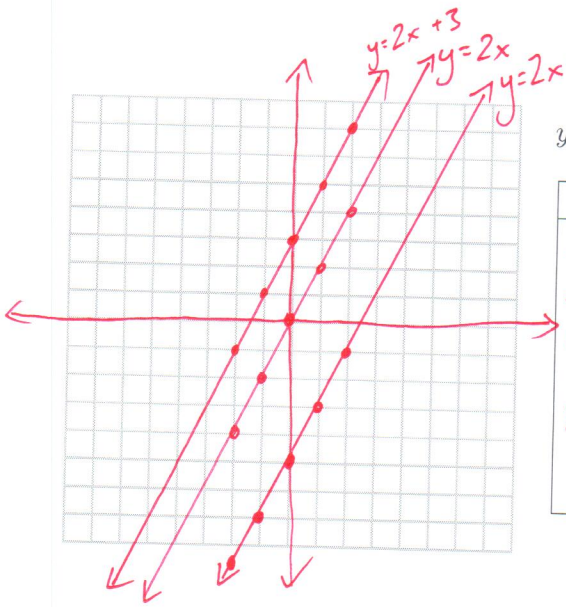
$-1.2 = -3$

X

None of the points are on the line $y = 2x - 1$

EXPLORATION 3

1. On the same coordinate plane, graph the lines given by each of the 3 equations below . [Optional: Use graphing calculator].



$y = 2x + 3$

x	y
-2	-1
-1	1
0	3
1	5
2	7

$y = 2x - 5$

x	y
-2	-9
-1	-7
0	-5
1	-3
2	-1

$y = 2x$

x	y
-2	-4
-1	-2
0	0
1	2
2	4

2. Comparing the graphs of each of these lines, what do you notice? What is similar? What is different?
They are parallel, they have the same slope.
The x-intercepts & y-intercepts are different, they are in different quadrants.

3. Can you come up with another parallel line to put on the graph?
Answers will vary. $y=2x-3$ or $y=2x+5$ may be common.

4. What do you notice about the equation of the line that you created and the equations of the lines that were given? What is the same? What is different?
Slope is the same, they are both in slope-intercept form.
y-intercepts are different.

5. What if you had the following equations? What can you say about their graphs? What is similar? What is different? How are these lines different from the ones above?

$y = -3x + 2$
$y = -3x - 1$
$y = -3x$

- steeper but sloping down
- negative slopes
- parallel
- different x and y-intercepts

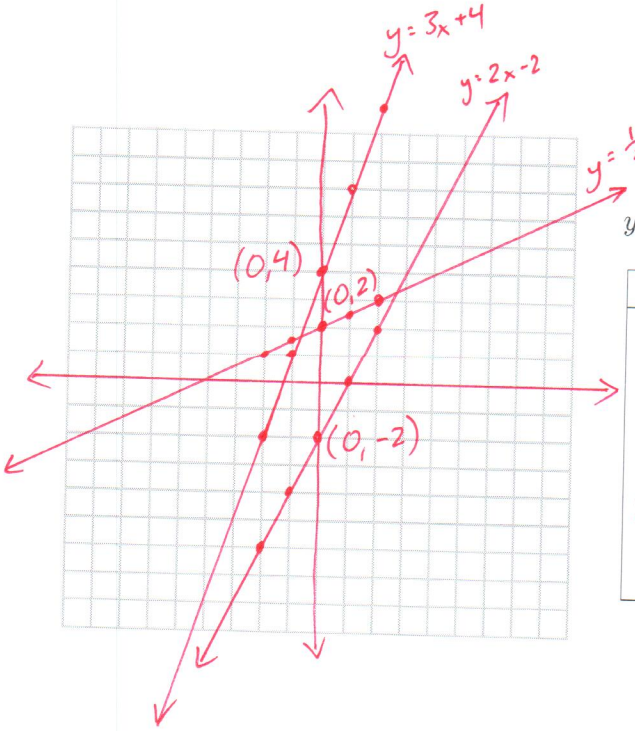
PROBLEM 2

The lines above have the form $y = mx + b$. Determine the values of m and b for each of the equations in Exploration 3. What does the number m tell you about the graph of the line? What does b tell you about the graph of the line? m is the slope (steepness) and whether it goes up or down. b is the y-intercept (where the line crosses the y-axis).

$y = 2x + 3$	$y = 2x - 5$	$y = 2x$	$y = -3x + 2$	$y = -3x - 1$	$y = -3x$
$m = 2$	$m = 2$	$m = 2$	$m = -3$	$m = -3$	$m = -3$
$b = 3$	$b = -5$	$b = 0$	$b = 2$	$b = -1$	$b = 0$

EXPLORATION 4

Graph each of the lines determined by the equations below. For each line, find the slope m and identify and label the point where the line crosses the y-axis.



$y = 2x - 2$, $m = 2$

x	y
-2	-6
-1	-4
0	-2
1	0
2	2

$y = 3x + 4$, $m = 3$

x	y
-2	-2
-1	1
0	4
1	7
2	10

$y = \frac{1}{2}x + 2$, $m = \frac{1}{2}$

x	y
-2	1
-1	1.5
0	2
1	2.5
2	3

EXAMPLE 1

In Exploration 1, the straight line goes through the points $(2, 3)$ and $(5, 9)$. We have determined the y-intercept by looking at the graph. Now determine the y-intercept by finding b in the equation of the line instead.

slope: $m = 2$

$y = 2x + b$

one point is $(2,3)$ (and $(5,9)$ can be used instead)

$(3) = 2(2) + b$

$3 = 4 + b$

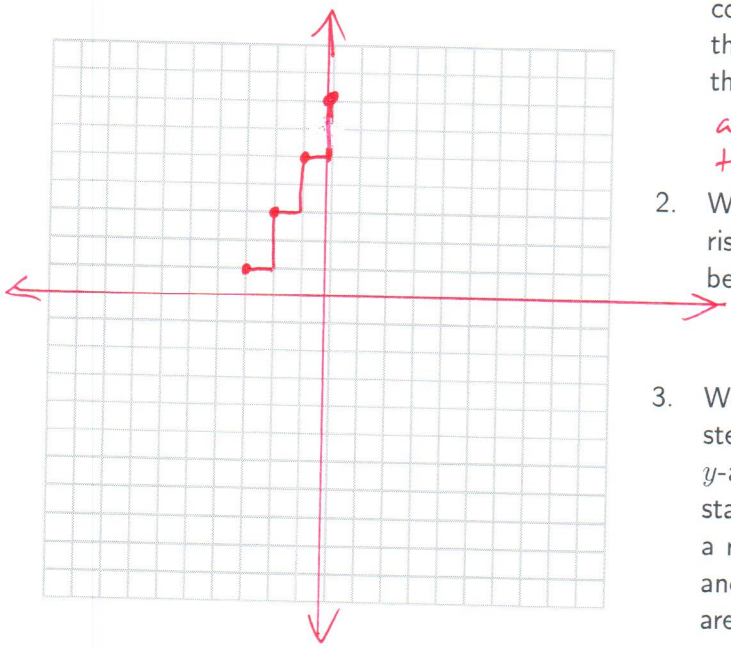
$3 - 4 = 4 + b - 4$

$-1 = b$

intercept: $(0, -1)$

EXPLORATION 5

Now we explore a graphical method for finding the y-intercept of the line which has slope $m = 2$ and goes through point $(-3, 1)$.



1. Plot the point $(-3, 1)$ on a coordinate grid. Explain why the the y-intercept must be greater than 1. The y-value goes up as we move right because the slope is positive
2. Write the slope as a ratio of the rise and the run. What will the rise be for a run$= 1$?

   $2 = \frac{\text{rise}}{\text{run}}$ rise $= 2$ when run $= 1$
3. We will use "slope" triangles to step from the point towards the y-axis. Make a "slope triangle" starting at the point $(-3, 1)$ with a run $= 1$. This triangle leads to another point on the line. What are its coordinates?

   $(-2, 3)$
4. Continue drawing triangles, each with run$= 1$ until you reach the y-axis. What are the coordinates of the point where the line crosses the y-axis? What is the equation of the line?

   $(0, 7)$ $y = 2x + 7$

EXAMPLE 2

Consider the two points $(1, 5)$ and $(2, 9)$. Find the slope, y-intercept and the equation of the straight line containing these two points.

slope $= \frac{\text{rise}}{\text{run}} = \frac{9-5}{2-1} = \frac{4}{1} = 4$

one method: $y = mx + b$

$(5) = 4(1) + b$

$5 - 4 = 4 + b - 4$

$b = 1$

another method:

$y - 5 = 4(x - 1)$

$y - 5 = 4x - 4$

$y - 5 + 5 = 4x - 4 + 5$

$y = 4x + 1$ so $b = 1$

$y = 4x + 1$

PROBLEM 3

Suppose a line has slope $\frac{1}{2}$ and contains the point $(3, 4)$. What is the y-intercept of this line? What is an equation for this line?

$y = mx + b$

$(4) = \frac{1}{2}(3) + b$

$4 = \frac{3}{2} + b$

$4 - \frac{3}{2} = \frac{3}{2} + b - \frac{3}{2}$

$b = \frac{8}{2} - \frac{3}{2}$

$b = \frac{5}{2}$

$y = \frac{1}{2}x + \frac{5}{2}$

EXAMPLE 3

Consider the line which passes through the two points $(1, 5)$ and $(2, 9)$. Use the formula for the slope and an arbitrary point (x, y) on the line to find the equation of the line.

1. Compute the slope $m = \frac{y_2 - y_1}{x_2 - x_1} =$ $\frac{9-5}{2-1} = \frac{4}{1} = 4$ or $\frac{5-9}{1-2} = \frac{-4}{-1} = 4$ $m = 4$

2. Let (x, y) be a point on the line. Explain why $m = \frac{y-5}{x-1}$ must be true.

If (x_1, y_1) is $(1, 5)$ and (x_2, y_2) is (x, y) then $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y-5}{x-1}$

3. Multiply both sides of the equation $m = \frac{y-5}{x-1}$ to get the point-slope form of the line.

$(x-1)m = \frac{(y-5)}{(x-1)}(x-1)$

$y - 5 = m(x-1)$

$y - 5 = 4(x-1)$

4. What happens if we start with $m = \frac{y-9}{x-2}$? Why does this represent the same line? *Hint: Put both equations in slope intercept form.*

$(x-2)\frac{y-9}{x-2} = m(x-2)$

$y - 9 = m(x-2)$

$y - 9 = 4(x-2)$

$y - 9 = 4x - 8$

$y - 9 + 9 = 4x - 8 + 9$

$y = 4x + 1$

$y - 5 = 4(x-1)$

$y - 5 + 5 = 4x - 4 + 5$

$y = 4x + 1$

same equation

PROBLEM 4

A line has slope $\frac{1}{3}$ and passes through $(2, 1)$.

1. Write the equation of the line in point-slope form.

$y - 1 = \frac{1}{3}(x - 2)$

2. Write the equation of the line in slope-intercept form.

$y - 1 = \frac{1}{3}x - \frac{2}{3}$ $+1$ $+1$

$y = \frac{1}{3}x - \frac{2}{3} + 1$ → $y = \frac{1}{3}x - \frac{2}{3} + \frac{3}{3}$

$y = \frac{1}{3}x + \frac{1}{3}$

3. What is the y-intercept?

$b = \frac{1}{3}$

SUMMARY (What I learned today)