FRACTIONS 4

Name: Key **Date:**_______ **Period:**______

SECTION 4.3 Unit Fractions, Mixed Fractions, Proper and Improper Fractions

VOCABULARY

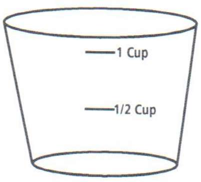
DEFINITION	EXAMPLE
Unit Fraction: A fraction with a numerator of 1 and a positive denominator	$\frac{1}{3}, \frac{1}{17}, \frac{1}{2}, \frac{1}{5}$
Mixed Fraction or Mixed Number: an integer with a fractional part (less than 1)	$2\frac{1}{8}, 3\frac{6}{7}$
Improper Fraction: a fraction greater than 1	$\frac{9}{2}, \frac{7}{6}, \frac{3}{2}$
Proper Fraction: a fraction less than 1	$\frac{2}{9}, \frac{1}{6},$

Big Idea: How are mixed fractions and improper fractions related?

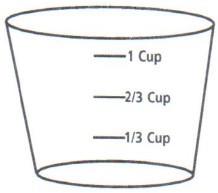
EXPLORATION 1: INTRODUCTION TO MIXED NUMBERS AND IMPROPER FRACTIONS

Recall on the Fraction Chart in Section 4.2 we have $\frac{1}{2}+\frac{1}{2}=1, \frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1.$

Notice that two halves equal 1

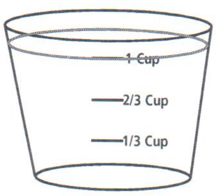


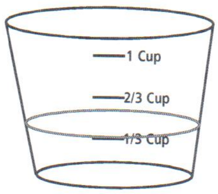
and three-thirds equal 1



What does $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ equal?

$\frac{4}{3}$ or $1\frac{1}{3}$ cups





A **unit fraction** always has 1 in the numerator. The denominator is a positive integer.

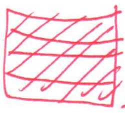
For example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and so on are all examples of unit fractions.

Now extend the addition of unit fractions to make another connection to multiplication. You have seen several models that represent the fraction $\frac{3}{5}$. In the area model, $\frac{3}{5}$ represents three $\frac{1}{5}$'s of a whole. This means $\frac{3}{5}$ is the sum of $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$. In the frog model, this is the same as taking 3 jumps of length $\frac{1}{5}$. That is, $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{1}{5}\cdot 3=\frac{3}{5}$. This understanding can be extended to all fractions. For example, the fraction $\frac{5}{9}$ is the same as the sum of 5 copies of $\frac{1}{9}$:

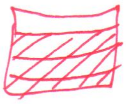
$$\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}=\frac{1}{9}\cdot 5=\frac{5}{9}.$$

EXAMPLE 1

Let's say you are going to bake cookies and the recipe calls for $1\frac{3}{4}$ cups of sugar. You look throughout the kitchen and the only measuring cup you can find is the $\frac{1}{4}$ c. measuring cup. How can you use that measuring cup to measure the $1\frac{3}{4}$ cups of sugar you need? Draw a picture and record your answer below.



$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$



$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$

You will need ___7___ $\frac{1}{4}$-cups.

EXAMPLE 2

Look at the mixed fraction $2\frac{1}{4}$. If you have only a quarter-cup measure, describe how you can measure the correct amount with the quarter cup.

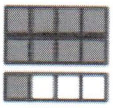
4 quarter-cups for each cup, and one more quarter cup. So, 4 + 4 + 1 is 9, or

$2\frac{1}{4}=\frac{9}{4}$

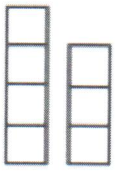
$\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)+\frac{1}{4}$

Did you find $2\frac{1}{4}$ equivalent to $\frac{9}{4}$? In fact, what you have found are two ways to write the same quantity: as a **mixed fraction**, $2\frac{1}{4}$, and as an **improper fraction**, $\frac{9}{4}$. How would you describe improper fractions? Why do you think they are called improper?

represent a quantity greater than 1 whole

$\frac{9}{4}$  $2\frac{1}{4}$

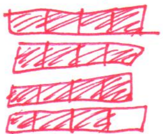
If we have seven quarters, we can think of each quarter as $\frac{1}{4}$ of one dollar. Then we have $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$. Because each of the four quarters equals one dollar, you have 1 dollar and 3 more quarters or $1\frac{3}{4}$ is equal to $\frac{7}{4}$.



Use a model and repeated addition sentences to find the improper fraction for each of the following mixed fractions:

a. $1\frac{7}{8}$ $\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)+\frac{1}{8}+\frac{1}{8}$ $+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ $=\frac{15}{8}$

b. $3\frac{3}{4}$ $\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)$ $+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{15}{4}$



EXAMPLE 3

Another way to think about $\frac{7}{4}$ is to view this fraction as a division problem, 7 divided by 4. If we divide 7 by 4, we have a quotient of 1 with a remainder of 3. Using the area model, we group 4 of the 7 into a rectangle of dimension 4 by 1 because 4 is the divisor. The remaining 3 fill up $\frac{3}{4}$.

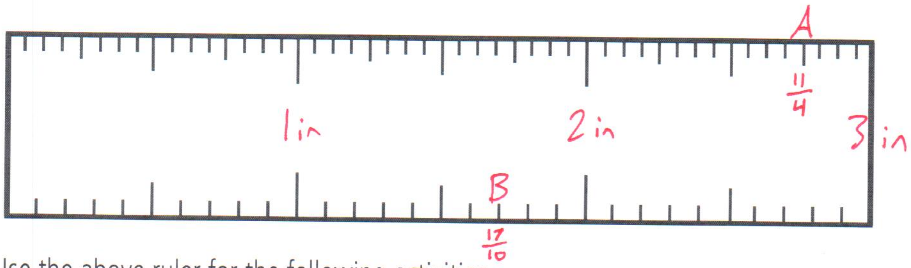
$\frac{7}{4}$  $1\frac{3}{4}$

$$\frac{7}{4}=7\div 4=1\frac{3}{4}$$

Complete the table below. The first one is done for you.

IMPROPER FRACTION	DIVISION PROBLEM	MIXED FRACTION
$\frac{11}{6}$	$11 \div 6$	$1\frac{5}{6}$
$\frac{53}{12}$	$53 \div 12$ $53 = 4(12) + 5$	$4\frac{5}{12}$
$\frac{73}{10}$	$73 \div 10$ $73 = 7(10) + 3$	$7\frac{3}{10}$

EXPLORATION 2: RULER ACTIVITY



Use the above ruler for the following activities.

1. The ruler represents 3 inches. Label the marks representing 1 inch, 2 inches, and 3 inches.
2. How many equal pieces is the top scale divided into? What should we call these pieces?
   16 in 1 inch
   48 in 3 inches
   sixteenths
3. How many equal pieces is the bottom scale divided into? What should we call these pieces?
   10 in 1 inch
   30 in 3 inches
   tenths
4. Label the tick marks for each scale.
5. Which tick mark would be equal to a length of $\frac{11}{4}$? Label this mark with an *A*. What is the mixed fraction name for this length? $2\frac{3}{4}$ inches
6. How many $\frac{1}{16}$ inches are in $2\frac{1}{4}$ inches? 36
7. How many $\frac{1}{8}$ inch are in $2\frac{1}{4}$ inches? 18
8. Write two other ways to say $\frac{2}{3}$ inches. $\frac{4}{6}$ inches, $\frac{6}{9}$ inches, etc.

9. Which tick mark on the bottom scale would be equal to a length of $\frac{17}{10}$? Label this mark with a B. What is the mixed fraction name for this length? What is the decimal name for this length?

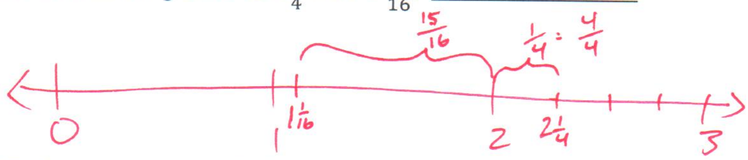
$1\frac{7}{10}$ inches   1.7 inches

10. Comparing the two scales, which number is greater: $\frac{11}{8}$ inches or $\frac{14}{10}$ inches? What are the mixed fraction names for these lengths?

$\frac{14}{10}$ inches is greater.

$\frac{11}{8} = 1\frac{3}{8}$ inches $\frac{14}{10} = 1\frac{4}{10} = 1\frac{2}{5}$ inches

11. How much greater is $2\frac{1}{4}$ than $1\frac{1}{16}$? $\frac{19}{16} = 1\frac{3}{16}$



12. If you had been told the above ruler was 6 inches long, how would that change the scale? In other words, how would that have changed the way you marked the tick marks?

each tick mark would be $\frac{1}{8}$ on top and $\frac{1}{5}$ on bottom

insted of $\frac{1}{16}$ and $\frac{1}{10}$

PROBLEMS:

1. Rewrite each sum as an improper fraction and as a mixed number.

UNIT FRACTIONS	IMPROPER FRACTION	MIXED NUMBER
$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=$	$\frac{7}{3}$	$2\frac{1}{3}$
$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=$	$\frac{8}{5}$	$1\frac{3}{5}$
$\frac{1}{2}+\frac{1}{2}=$	$\frac{2}{2}$	1
$\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=$	$\frac{9}{7}$	$1\frac{2}{7}$

2. How many one-sixth cups of sugar are in $1\frac{2}{3}$ cups of sugar? 10

$1 = \frac{6}{6}$ $\quad$ $\frac{2}{3} = \frac{4}{6}$

3. Which is greater, $2\frac{5}{8}$ or $\frac{30}{16}$? $2\frac{5}{8}$ Explain your reasoning.

$\frac{30}{16} = 1\frac{14}{16} = 1\frac{7}{8}$

$2\frac{5}{8} = \frac{16}{16} + \frac{16}{16} + \frac{10}{16} = \frac{42}{16}$

$2\frac{5}{8}$ contains more sixteenths than $\frac{30}{16}$ does.

4. Each slice of pizza is $\frac{1}{8}$ of a pizza. You and your friends are to get 3 slices each. If you have 6 pizzas, how many people can be served?

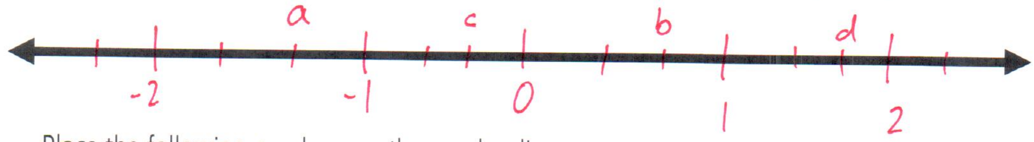
6 pizzas $\rightarrow$ $6 \cdot 8$ slices $\rightarrow$ $\frac{\text{48slices}}{\text{3 per person}}$

16 people can be fed.

5. Mrs. Miranda cut construction paper into fifths to pass out to her students to make bookmarks. If she passed out $\frac{90}{5}$ how many pieces of construction paper were in the box to begin with?

$\frac{90}{5} = 90 \div 5 = 18$ pieces of paper

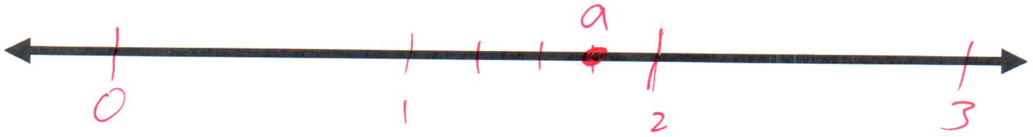
6. Place zero in the center of the number line below. Number by thirds from $-2\frac{1}{3}$ to $2\frac{1}{3}$.



Place the following numbers on the number line.

a. $-\frac{8}{6} = -\frac{4}{3}$ $= -1\frac{1}{3}$ b. $\frac{8}{12} = \frac{2}{3}$ c. $-\frac{4}{12} = -\frac{1}{3}$ d. $\frac{5}{3} = 1\frac{2}{3}$

7. Use the number line below to plot points in the following problems.



a. A recipe for pancakes calls for $1\frac{3}{4}$ cups of flour. Locate this point on the number line above. Describe the equivalent improper form for the mixed fraction. What does the numerator represent?

$\frac{7}{4}$ numerator is the number of quarter cups.

b. Jack has 3 identical pans of brownies and decides to divide each pan into 12 equal pieces. How many brownies pieces does he have in all? 36 brownie pieces

12 + 12 + 12

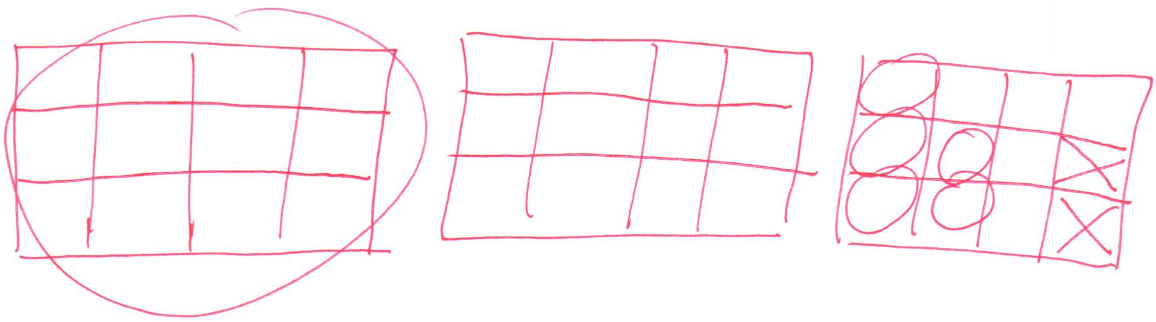
Because Jack was very hungry, he ate 2 of the pieces. If you assume each brownie pan represents 1 or a whole, express the amount of brownies that remains in terms of the whole and pieces.

$\frac{34}{12} = \frac{12}{12} + \frac{12}{12} + \frac{10}{12} = 2\frac{5}{6}$

c. If Jack takes half of the uneaten brownies to a party, what quantity will he take? $1\frac{5}{12}$

17 brownies, or $\frac{17}{12}$ wholes $= \frac{12}{12} + \frac{5}{12}$

Using the area model, draw the quantities of brownies that he will take to the party. Be sure to include the fact that each pan is divided into 12 pieces.



SUMMARY (What I learned in this section)