MULTIPLICATION AND DIVISION 4

Name: Key Date:________ Period:______

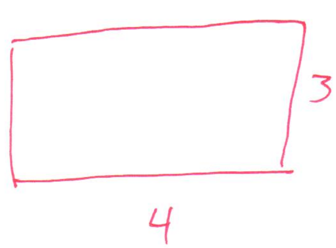
SECTION 4.2 AREA MODEL FOR MULTIPLICATION

VOCABULARY

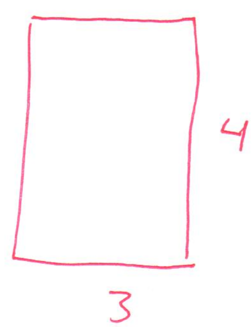
DEFINITION	EXAMPLE
Area model: Representation of multiplication through area	3 2 $\boxed{}$ $2 \cdot 3 = 6$
Commutative Property of Multiplication: for numbers A and B, $A \cdot B = B \cdot A$	$4 \cdot 3 = 3 \cdot 4$ $12 = 12$
Associative Property of Multiplication: $(xy)z = x(yz)$ The order numbers are multiplied does not matter	$(2)(3)(6)$ $(2 \cdot 3)6 = 2 \cdot (3 \cdot 6)$ $6 \cdot 6 = 2 \cdot 18$ $36 = 36$
Distributive Property of Multiplication Over Addition: For any numbers k, m, and n, $n(k+m) = nk + nm$	$3(2+1) = 3(2) + 3(1)$ $3 \cdot 3 = 6 + 3$ $9 = 9$

Big Idea: How do we use area models to represent multiplication? What properties can we use in multiplication?

Draw a rectangle with a width of 3 units and a length of 4 units. What is the area of the rectangle? How does this compare to a rectangle with a width of 4 units and a length of 3 units? How does this example relate to the commutative property of multiplication?



$4 \cdot 3 = 12$



$3 \cdot 4 = 12$

If we were to multiply three numbers together, does the order in which we multiply matter? For example, is the product of $(2 \cdot 3) \cdot 4$ the same as the product of $2 \cdot (3 \cdot 4)$? In this example, we change the grouping of the numbers. Relate this example to the associative property of multiplication.

No, order does not matter.

$(2 \cdot 3)4 = 2(3 \cdot 4)$
$6 \cdot 4 = 2 \cdot 12$
$24 = 24$

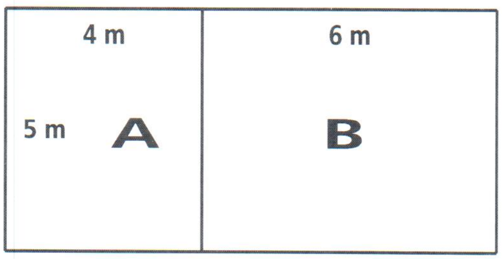
Associative property of multiplication:
$(xy)z = x(yz)$

EXAMPLE 1

The Elliot's are constructing a small building that is one room wide and two rooms long. Each room is 5 meters wide. The front room (A) is 4 meters long and the back room (B) is 6 meters long. What is the floor space of each room? What is the floor space of the building? How are the areas of the two rooms related to the area of the building? The floor plan below shows the situation:

Area of room A = (5m)(4m) = 20 m^2

Area of room B = (5m)(6m) = 30 m^2



5m(10m) = 50m^2
4m+6m

sum of the two rooms

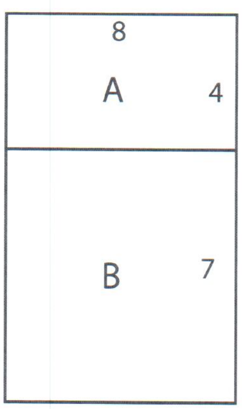
Total Area = (5m)(4m) + (5m)(6m) = 50m^2
20m^2 + 30m^2 = 50m^2

PROBLEM 1

Compute the area of the larger rectangle by computing the areas of the inner rectangles.

area of Rectangle A = (8u)(4u) = 32 sq units

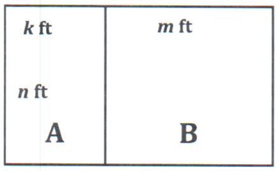
area of Rectangle B = (8u)(7u) = 56 sq. units



Total Area = (8u)(4u) + (8u)(7u) = 32sq. u. + 56 sq. u
88 sq. units

EXAMPLE 2

Now suppose the dimensions of the Elliot's building have not been decided yet. We need a formula for the areas. Call the width of the building n feet and the lengths of rooms 1 and 2, k and m feet respectively. Find the area of each room and the building's total area.



Area of room A = (n ft) (k ft) = $n \cdot k \ ft^2$

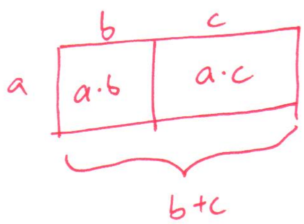
Area of room B = (n ft) (m ft) = $n \cdot m \ ft^2$

Total Area = (n ft) (k ft) + (n ft) (m ft) = $nk + nm \ ft^2$

Is there another way you can write the equation for total area? $n \ ft (k \ ft + m ft)$

PROBLEM 2

Use the distributive property to write the following product as a sum: a (b + c). Draw a picture with rectangles to illustrate this property.

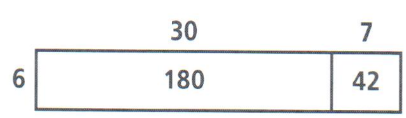


$\boxed{a \cdot b + ac} = a(b+c)$

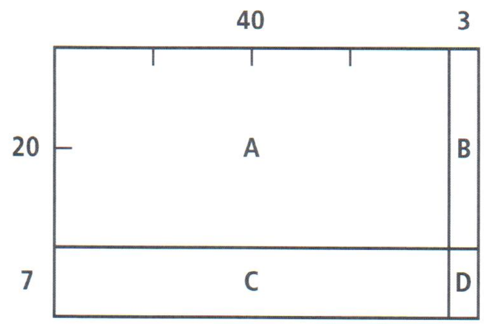
Application of Distributive Property

You have already learned to multiply two-digit and three-digit numbers. Now you can use the area model and the distributive property to explore this process carefully. Begin by modeling the product of a one-digit number and a two-digit number. To multiply 6 · 37, use place value to write the product 6 · 37 as 6 (30+7).

By the distributive property, 6 · 37 = 6 (30 + 7) = 6 · 30 + 6 · 7 = 180 + 42 = 222.



Visualize the product of 27 x 43 as area with the picture below:



Area of A = $20 \cdot 40$ = 800;
Area of B = $20 \cdot 3$ = 60;
Area of C = $7 \cdot 40$ = 280;
Area of D = $7 \cdot 3$ = 21. The total area is 800 + 60 + 280 + 21 = 1161.

We can see this in our vertical format below:

$$\begin{array}{r} 27 \\ \underline{\times 43} \\ 21 \\ 60 \\ 280 \\ \underline{+800} \\ 1161 \end{array}$$

You can extend the same process to multiply 27 by 43 using the distributive property

$$\begin{aligned} 27 \cdot 43 &= (20 + 7)(40 + 3) \\ &= 20(40 + 3) + 7(40 + 3) \\ &= 20 \cdot 40 + 20 \cdot 3 + 7 \cdot 40 + 7 \cdot 3 \\ &= \underline{800} + \underline{60} + \underline{280} + \underline{21} = \underline{1161} \end{aligned}$$

EXAMPLE 3

Pat has a rectangle with a length that is 3 units more than *x*, a positive length. The width of the rectangle is 4 units. Draw a picture to represent this problem. What is the area of the rectangle?

EXAMPLE 4

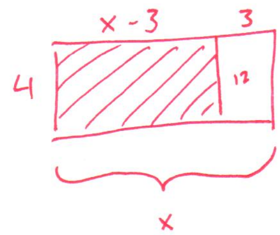
Luz sees an advertisement for \$3 off the regular price of DVDs in her favorite store. She buys 4 DVDs, all of which have the same regular price of more than \$3. Represent this situation with an algebraic expression and with an area model representation.

x = regular price

sale price = x − 3

algebraic: 4(x − 3)

4(x + (−3))



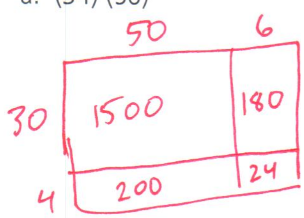
4x − 12 is the shaded area of 4(x − 3)

so $\boxed{4(x-3) = 4x - 12}$

PRACTICE EXERCISES

1. Use the area model and the distributive property to compute the following products. Indicate the area of each interior part in your model.

a. (34) (56)



sum is

34 · 56 = 1904

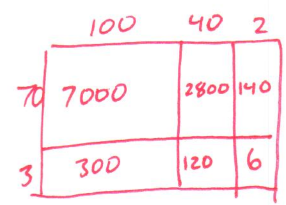
34(50 + 6)

34(50) + 34(6)

(30 + 4)50 + (30 + 4)6

30 · 50 + 4 · 50 + 30 · 6 + 4 · 6 = 1500 + 200 + 180 + 24 = 1904

b. (142)(73)



142 · 73 = 10366

142(70 + 3)

142(70) + 142(3)

100 · 70 + 40 · 70 + 2 · 70 + 100 · 3 + 40 · 3 + 2 · 3

7000 + 2800 + 140 + 300 + 120 + 6 = 10366

2. Use the distributive property to simplify the following expressions. This is also called **combining like terms**.

a. 3(4) + 3(2)

b. 2x + 5x

c. 6a – 3a

d. -2y – 6y

3(4 + 2)
3 · 6
18

x(2 + 5)
x · 7
7x

a(6 − 3)
a · 3
3a

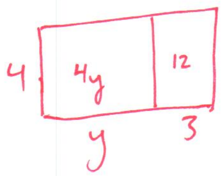
−y(2 + 6) or y(−2 − 6)
−y · 8   y · (−8)
−8y   −8y

3. Draw area model using rectangles for the following problems and then compute the following products using the distributive property.

a. $4(y + 3)$

$4y + 4 \cdot 3$

$\boxed{4y + 12}$



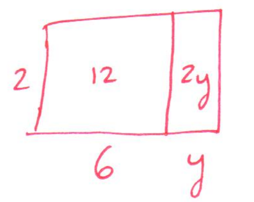
$4y + 12$

b. $2(6 + y)$

$2 \cdot 6 + 2y$

$12 + 2y$

$\boxed{2y + 12}$



SUMMARY (What I learned today)