SECTION 3.5 FUNCTIONS VS. EQUATIONS

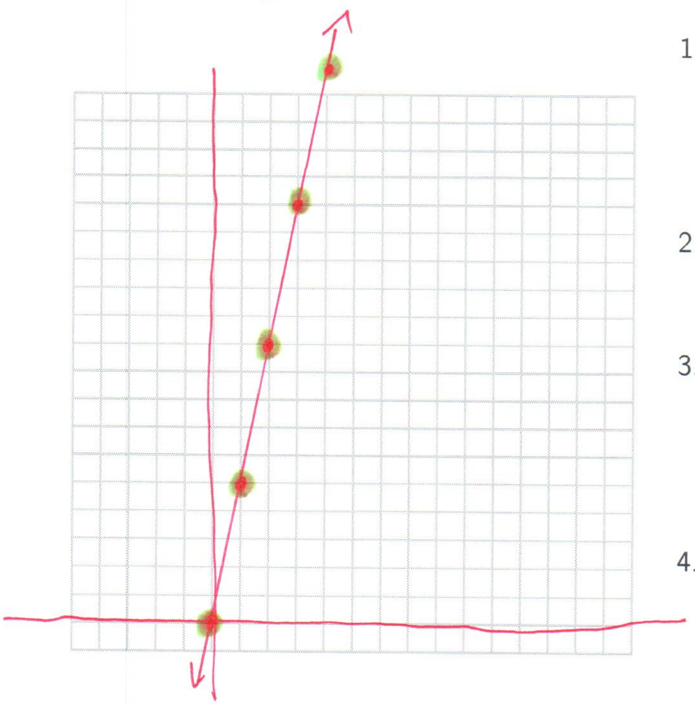
Name: Key **Date:** ________ **Period:** _______

Vocabulary

DEFINITION	EXAMPLE
x-intercept The value of x when the function equals 0, x-coordinate where a line crosses the x-axis	$f(x) = 2x + 1$ $0 = 2x + 1$ $x = \frac{-1}{2}$
Fixed Cost Initial value when y-intercept is not 0.	money to set up a lemonade stand before sales begin.
Variable Cost Amount of cost that increases as x gets larger.	increase in money earned as more glasses of lemonade are sold.

EXAMPLE 1

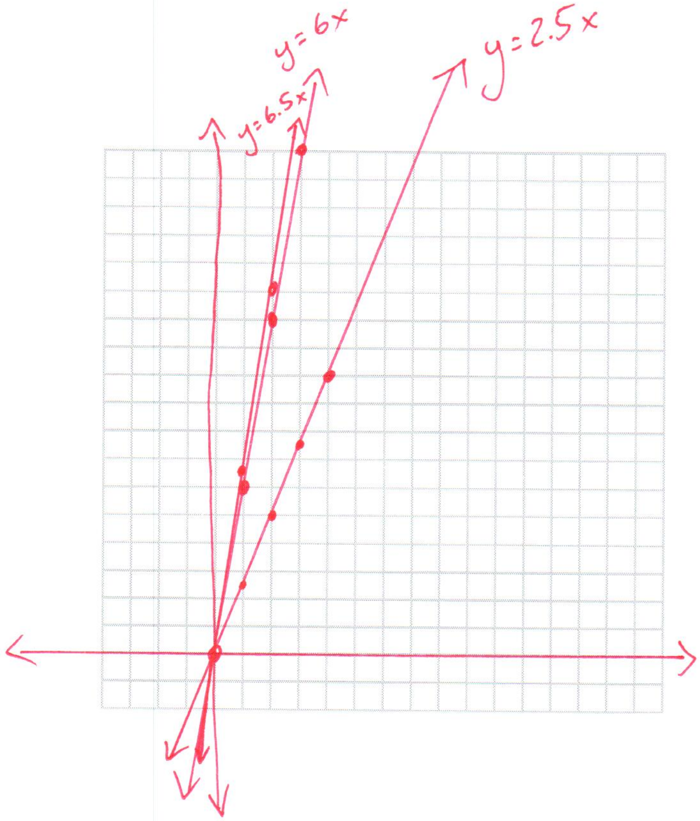
Susan has a bag of nickels with x nickels.



1. Determine the formula for the linear function describing the value, $V(x)$ and specify its domain.
   $V(x) = 5x$ where $V(x)$ is the value in cents. domain is whole numbers.
2. Make a graph of the function.
   key: • • etc. (discrete)
3. Find the equation of the line that passes through the points of the graph of the function.
   $y = 5x$ $m = \frac{5-0}{1-0} = 5$ $b = 0$
4. Graph the line you found in part 3. How is the graph of this line different from the graph of the function? How is it the same?
   key: ↔ line contains more points, including fractional coordinates (continuous)

PROBLEM 1

For each of the following, determine the formula for the linear function, specify its domain and sketch a graph of the function. Also find the corresponding equation of the corresponding line and sketch the line.



1. Each baseball costs $6. What is the formula for the cost $C(x)$ of x baseballs?

   C(x)=6x   y=6x

   domain is whole numbers
   domain is discrete

2. A movie theater charges $6.50 for the latest blockbuster. How much does it cost a group of x friends to see the movie? (What is the function?)

   C(x)=6.5x   domain = whole numbers (discrete)

   y=6.5x

3. Ramon's pet snail can crawl 2.5 centimeters per minute. What is the formula for the distance $d(x)$ that the snail can crawl in x minutes?

   d(x)=2.5x

   domain is x≥0 (continuous)

   y=2.5x

As you can see in Problem 1, we can use either the function approach or equation approach. Furthermore, they are very similar. So why do we have two approaches? Each approach has its advantages. Consider Susan's bag of nickels described in Example 1.

Advantages of the function approach:

- The label of the function V reminds us that the function represents the value of the nickels.
- The function notation $V(x)$ makes it clear that we are thinking of the number of nickels x as the independent variable (input) and the value $V(x)$ as the dependent variable (output) .
- In the function approach, the domain (set of inputs) must be specified. In many applications, including this one, restrictions on the domain are very important. For example, Susan can only have a whole number of nickels. On the other hand, in Problem 3 Ramon's snail can crawl for $x = \frac{1}{2}$ or $x = \frac{2}{3}$ minutes, but x can not be negative. Notice how this makes the graph of the function different from the graph of the line. However, the graph of the line can help us see the pattern of the function.

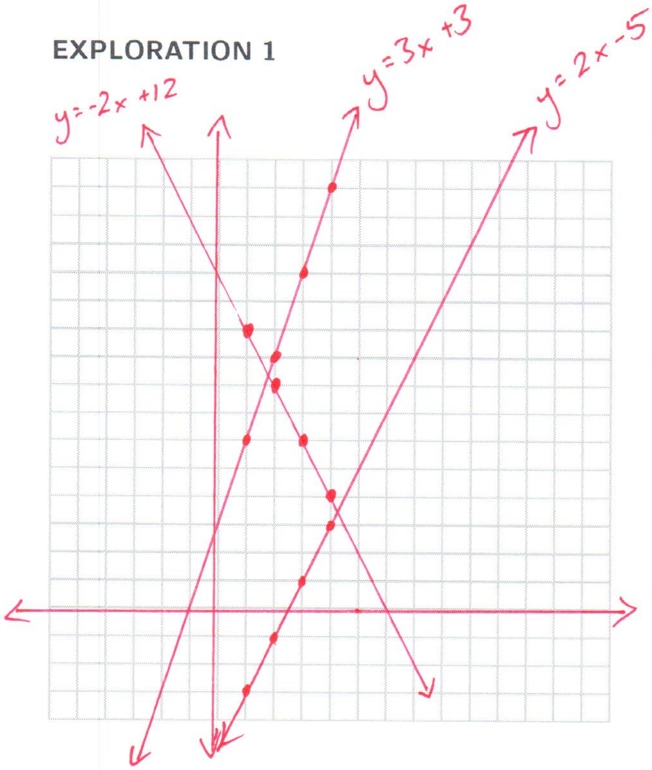
Advantages of the equation approach:

- There are different, but equivalent, forms of the equation of a line, for example, *slope-intercept* and *point-slope*. These different forms give more flexibility in manipulating the equations of the line.
- The geometrical properties of the line, slope and intercepts, are easier to see in the equation of the line.

* Lines are always continuous, but they can help model discrete functions

Sequences, Lines and Linear Functions

EXPLORATION 1



- $\{6, 9, 12, 15, \ldots\}$ 3 3 3
  Constant Difference: 3
  m=3 b=3
  Equation of Line: $y = 3x + 3$
- $\{10, 8, 6, 4, \ldots\}$ -2 -2 -2
  Constant Difference: -2
  m= -2 b=12
  Equation of Line: $y = -2x + 12$
- $\{-3, -1, 1, 3, \ldots\}$ 2 2 2
  Constant Difference: 2
  m=2 b=-5
  Equation of Line: $y = 2x - 5$

For each arithmetic sequence above:

1. Find the constant difference c.
2. Graph the sequence.
3. Notice that the points lie on a line. Draw the line through the graph.
4. Find the slope and y-intercept of the line. Write the equation of the line.
5. What role does the constant difference c play in the equation.

   It is the slope
6. It sometimes useful to think about a 0-th term. If the function form of the sequence is $a_n = A(n)$, then $a_0 = A(0)$. We can find a_0 by using the pattern to work backwards. What term should come before the first term? How does the 0-th term compare to the y-intercept? How does the first term a_1 compare to the y-intercept?

   3, 12, -5 → 0th term is the y-intercept =b $a_1 = m + b$
7. How can you use the equation of the line to find the formula for the n-th term?

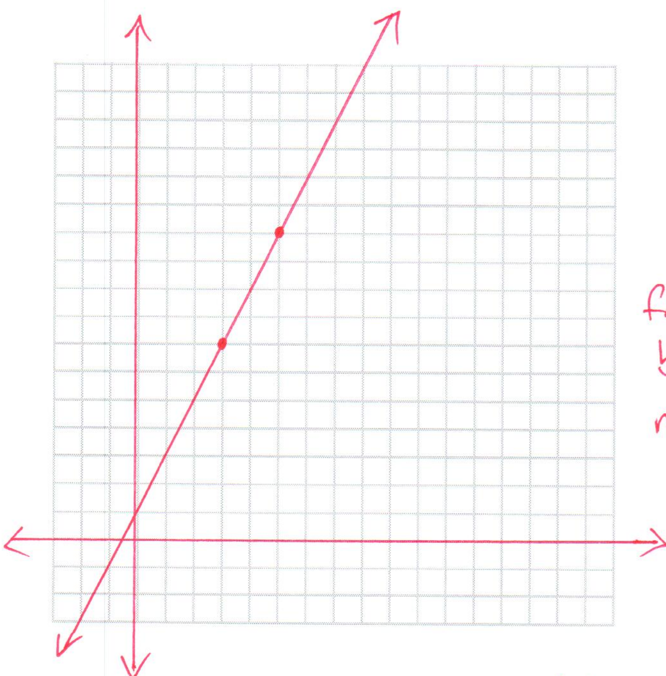
   n^{th} term $= a_n = m(n) + b$

PROBLEM 2

Two terms of an arithmetic sequence are given: $a_3 = 7$ and $a_5 = 11$. Write the first five terms of the sequence. Find the formula for the n-th term of the sequence.

Hint try the following steps:

1. Plot the points $(3, 7)$ and $(5, 11)$.
2. Since the sequence is arithmetic, the graph of the sequence should follow a line. Draw the line through the two points. Find the equation of the line.



from the graph:
$b=1$
$m=2$ $\quad y=2x+1$

$a_n=2n+1$

OR from the points:

$m=\frac{11-7}{5-3}=\frac{4}{2}=2$

$y=2x+b$ plug in a point

$(7)=2(3)+b$

$7=6+b$

$-6 \quad -6$

$1=b$

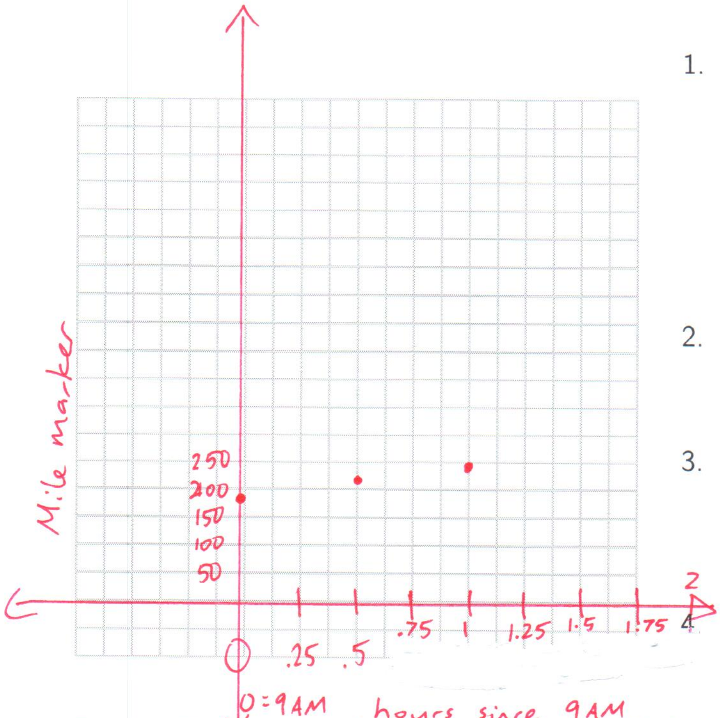
$y=2x+1$

$a_n=2n+1$

Rate of Change and Slope

EXPLORATION 2

A bus full of students is traveling down the freeway on a field trip. At 9:00 A.M., Carter looks out the window and sees the bus is passing mile marker 185. At 9:30 A.M., the bus passes mile marker 215. At 10:00 A.M., it passes marker 245. Carter wants to know how fast the bus is going.



1. What was the bus' average speed in miles per hour on the first leg of the trip from 9:00 to 9:30? From 9:30 to 10:00? Is the speed the same? $\frac{215-185}{30} = \frac{30}{30} = 1\ \frac{\text{mile}}{\text{minute}}$

   yes. $\frac{245-215}{30} = \frac{30}{30} = 1\ \frac{\text{mile}}{\text{minute}}$

2. Graph time versus distance. OR 60 mph

   scales may vary.

3. Draw a line through the graph. Find the equation of the line.

   $y = 60x + 185$

4. How does the speed found above compare with the slope?

   it is the same

5. Use the concept of slope to explain why, if the speed is constant, the graph of time versus distance will be a line.

   $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{miles}}{\text{hour}} = \text{speed}$ so the slope is constant and the graph is linear.

6. At 10:00 A.M., Carter asks the bus driver how much longer until they reach their destination. The bus driver says, "We will exit the freeway in 1 hour." If the speed is constant, how many more miles until the bus exits the freeway?

   speed = 60 miles per hour $\frac{y \text{ miles}}{1 \text{ hours}} = \frac{60 \text{ miles}}{1 \text{ hour}}$

   $y = 60$ miles

7. Speed is often referred to as the rate of change. Explain why this term makes sense.

   $\frac{\text{rise}}{\text{run}} = \frac{\text{change in miles}}{\text{change in time}}$ = rate of change of y with respect to x.

Unit Rate and Slope

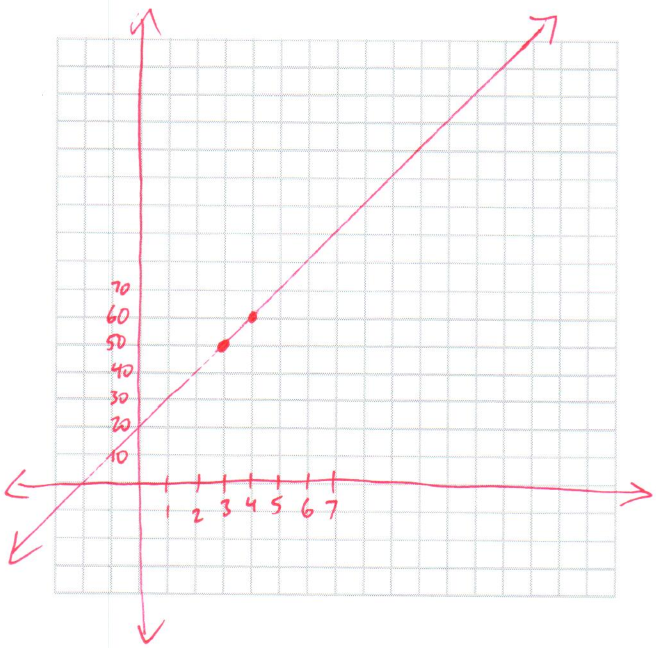
PROBLEM 3

Find the unit value for each function in Problem 1.

1) 6 dollars per baseball
2) 6.5 dollars per movie ticket
3) 2.5 centimeters per minute

EXPLORATION 3

Suppose that $y = C(x) =$ cost in \$ to produce x widgets, and that the graph of all points (x, y) that satisfy the cost equation fall on a straight line. Also, it costs \$50 to produce 3 widgets, and \$ 60 to produce 4 widgets.



1. Two points on the line are given in the problem. Identify and plot them. (3, 50) (4, 60)

2. Draw a line through the points.

3. What is the slope and y intercept of the line?

   $m = 10$ y-intercept $= b = 20$

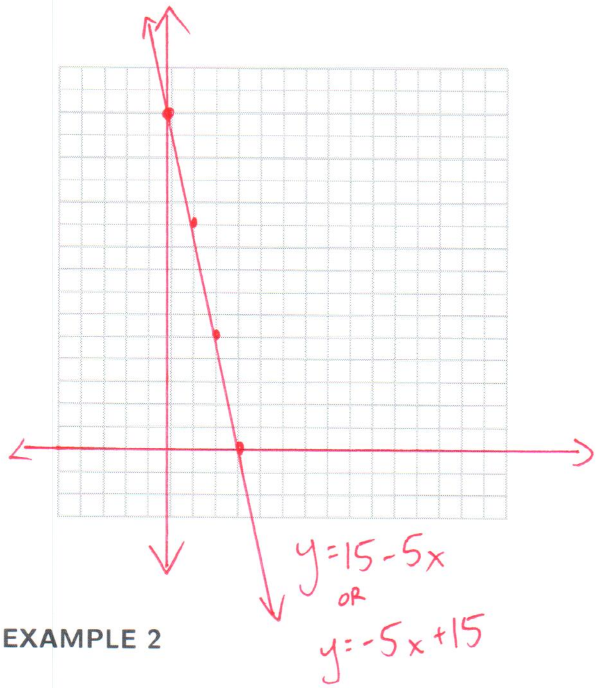
4. What is the unit cost of each widget?

   \$10

   since $m = \dfrac{\text{cost in \$}}{\text{\# of widgets}} = 10$

EXPLORATION 4

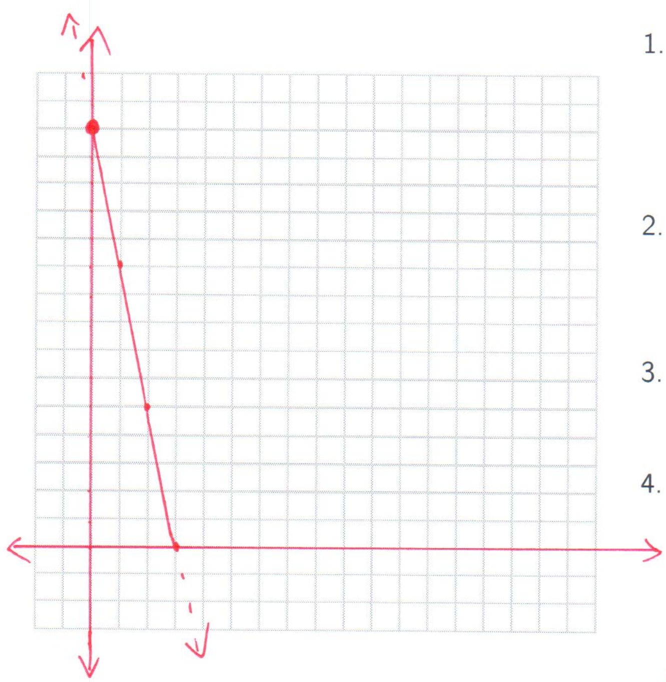
Consider the line given by the function $y = 15 - 5x$.



1. What is the slope and y-intercept of the line?
   m = -5 b = 15
2. Graph the line.
3. Find the x-coordinate of the point where the line crosses the x-axis. That is find x so that $(x, 0)$ is on the line. $(3, 0)$

EXAMPLE 2

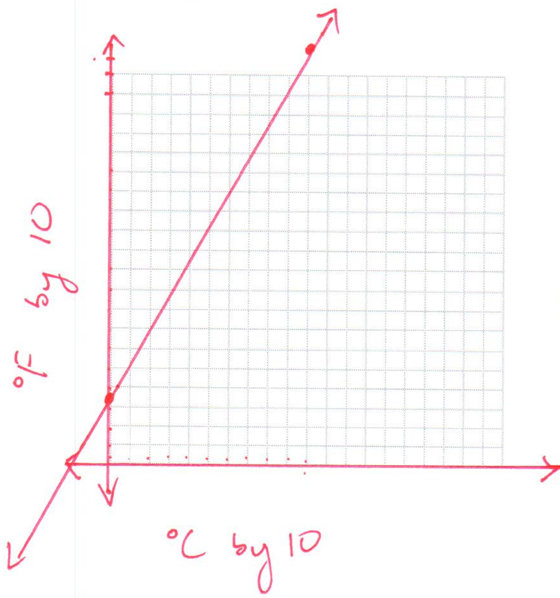
Juanita is 15 miles away from home. Suppose she walks at a constant rate of 5 miles per hour towards home for x hours.



1. Determine the formula for her distance in miles from home after walking for x hours, $D(x)$. What is the domain?
2. Sketch a graph of the function. Be careful, to think about what are possible inputs and outputs. solid only
   inputs: all non-negative outputs: all non-neg
3. Find and sketch the equation of the line $y = D(x)$. dashed and solid
4. Find and interpret the y and x intercepts of the line. What is the initial value and the terminal value for this problem? What do they represent? y-intercept: 15
   x-intercept: 3
   initial value: starting distance: 15 miles
   terminal value: ending time: 3 hours

EXAMPLE 3

The boiling point of water is 212°F and the freezing point is 32°F. Scientists wanted to create a more logical set of units. Using water as the standard they created the Celsius temperature scale, by letting 0°C represent the freezing point and 100 °C the boiling point. Let x be the temperature in Celsius, $F(x)$ the temperature in Fahrenheit and $y = F(x)$ the equation of the line.



1. Two points on the line are given, plot them. $(0, 32)$ freezing point $(100, 212)$ boiling point

2. Find the slope. What does it mean in this setting?

   $m = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$

   Fahrenheit temps change by $\frac{9}{5}$°F when celcius temps change by 1°.

3. Find the equation of the line.

   $y = \frac{9}{5}x + 32$

4. Find the x and y intercepts. Interpret them.

   y-intercept: $b = 32$ (freezing point of water)

   x-intercept: $0 = \frac{9}{5}x + 32$

   $\frac{5}{9}(-32) = \left(\frac{9}{5}x\right)\frac{5}{9}$

   $\frac{-160}{9} = x$

   0°F is the same as $\frac{-160}{9}$°C

5. Is the temperature ever the same in Fahrenheit and Celsius? If so when? If not explain.

   yes. Then $x = y$, so since the equation is $y = \frac{9}{5}x + 32$,

   $x = \frac{9}{5}x + 32$

   $x - \frac{9}{5}x = 32$

   $\frac{-5}{4}\left(\frac{-4}{5}x\right) = (32)\frac{-5}{4}$

   $x = -(8 \cdot 5)$

   $x = -40$ and $y = -40$

   So $-40°C = -40°F$

In this chapter, we have studied lines and linear functions. In Section 3.3 you explored proportional relationships. Now, we carefully investigate the difference between the concepts of linear and proportional.

	Proportional	Linear Non-proportional
The ratio $\frac{y}{x}$	constant	not constant
The rate of change $\frac{y_2-y_1}{x_2-x_1}$	constant	constant
The graph	line through origin	line with y-intercept $\neq 0$
The equation	$y = mx$	$y = mx + b$, $b \neq 0$

EXPLORATION 5

Think of different contexts from your previous math classes or from your experience outside of school. Identify 4 relationships, two that are proportional and two that are linear but not proportional. Explain how you can tell.

proportional	linear but not proportional
total cost of granola in terms of lbs of granola	money earned from lemonade stand, including set-up cost
distance travelled over time at a constant speed	Fahrenheit and Celcius conversion
(0,0) is on the graph, which is linear	y-intercept = b is not 0

EXAMPLE 4

In Exploration 3, the total cost in dollars of producing x widgets was $y = 10x + 20$.

1. Identify the fixed and variable cost.

   fixed cost: \$20

   variable cost: \$10x

2. Explain why the variable cost is proportional to the number of widgets produced.

   $\frac{10x}{x} = 10$, the unit cost to make a widget

3. Explain why the total cost is linear but not proportional to the number of widgets produced.

   b=20, since there is a start-up cost.

   The graph does not go through the origin.

   $\frac{y_2-y_1}{x_2-x_1}$ is constant but $\frac{y}{x}$ is not constant.

SUMMARY (What I learned today)