SECTION 3.6 STANDARD FORM OF A LINE AND ITS APPLICATIONS

Name: Key **Date:** ________ **Period:** ________

Vocabulary

DEFINITION	EXAMPLE
Standard Form equations written $Ax + By = C$ where $A, B, \& C$ are integers (if possible) and A is positive	$3x + 4y = 12$

PROBLEM 1

Rewrite each of the equations below in the slope-intercept form and identify its slope and y-intercept.

1. $3x + 4y = 12$

$3x + 4y - 3x = 12 - 3x$

$\frac{4y}{4} = \frac{-3x + 12}{4}$

$y = \frac{-3x}{4} + 3$

$m = \frac{-3}{4}$ $b = 3$

2. $-2x + y = 5$

$-2x + 2x + y = 2x + 5$

$y = 2x + 5$

$m = 2$ (slope)

$b = 5$ (y-intercept)

3. $2x - 5y = 6$

$2x - 2x - 5y = -2x + 6$

$\frac{-5y}{-5} = \frac{-2x}{-5} + \frac{6}{-5}$ $m = \frac{2}{5}$

$y = \frac{2}{5}x - \frac{6}{5}$ $b = \frac{-6}{5}$

4. $x + y = 4$

$x - x + y = 4 - x$

$y = -x + 4$

slope $= m = -1$

y-intercept $= b = 4$

5. $Ax + By = C$

$Ax - Ax + By = -Ax + C$

$By = -Ax + C$

$\frac{By}{B} = \frac{-A}{B}x + \frac{C}{B}$

slope $m = \frac{-A}{B}$ y-intercept $b = \frac{C}{B}$

So in general, the slope-intercept form is $y = \frac{-A}{B}x + \frac{C}{B}$

EXAMPLE 1

Convert the equation $y = \frac{2}{3}x + \frac{4}{3}$ into standard form: $Ax + By = C$ and identify what A, B and C are.

$3(y) = \left(\frac{2}{3}x + \frac{4}{3}\right)3$

$3y = 2x + 4$

$3y - 2x = 2x + 4 - 2x$

$-2x + 3y = 4$

$-(-2x + 3y) = (4)(-1)$

$2x - 3y = -4$

$Ax + By = C$

$A = 2 \quad B = -3 \quad C = -4$

PROBLEM 2

Convert each of the following equations into standard form:

1. $y = -4x + 6$

$y + 4x = -4x + 4x + 6$

$\boxed{4x + y = 6}$

2. $y = x - 3$

$y - x = x - 3 - x$

$-1(-x + y) = (-3)(-1)$

$\boxed{x - y = 3}$

3. $y = \frac{4}{3}x - 1$

$3(y) = \left(\frac{4}{3}x - 1\right)3$

$3y = 4x - 3$

$3y - 4x = 4x - 4x - 3$

$-4x + 3y = -3$

$-(-4x + 3y) = -(-3)$

$4x - 3y = 3$

4. $\frac{y}{2} = \frac{2}{3}x + \frac{1}{6}$

$\left(\frac{y}{2}\right)6 = \left(\frac{2}{3}x + \frac{1}{6}\right)6$

$3y = 4x + 1$

$-1(3y - 4x) = -1(1)$

$4x - 3y = -1$

EXPLORATION 1

We have seen how to compute the y-intercept. Apply the same approach to find the x-intercepts.

1. $y = 3x + 6$

$0 = 3x + 6$

$-6 = 3x$

$\frac{-6}{3} = \frac{3x}{3}$

$x = -2$

$(-2, 0)$

2. $2x + 5y = 8$

$2x + 5(0) = 8$

$2x = 8$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

$(4, 0)$

3. $y = 4$

$0 = 4$ ✗ ∴ There is no x-intercept since this is a horizontal line.

EXAMPLE 2

Compute the x-intercept and y-intercept for the line given by the equation: $6x + 5y = 30$.

x-intercept (where y = 0)

$6x + 5(0) = 30$

$6x = 30$

$x = 5$

$(5, 0)$

y-intercept (where x = 0)

$6(0) + 5y = 30$

$5y = 30$

$y = 6$

$(0, 6)$

PROBLEM 3

Compute the x-intercept and y-intercept for each of the lines given by the equations:

1. $4x + 3y = 12$

$4(0) + 3y = 12$

$3y = 12$

$y = 4$

y-intercept

$4x + 3(0) = 12$

$4x = 12$

$x = 3$

x-intercept

$(3, 0)$ and $(0, 4)$

2. $2x - 5y = 10$

x-intercept

$2x - 5(0) = 10$

$2x = 10$

$x = 5$

$(5, 0)$

y-intercept

$2(0) - 5y = 10$

$-5y = 10$

$y = -2$

$(0, -2)$

3. $3x + 7y = 12$

x-intercept

$3x + 7(0) = 12$

$3x = 12$

$x = 4$

$(4, 0)$

y-intercept

$3(0) + 7y = 12$

$7y = 12$

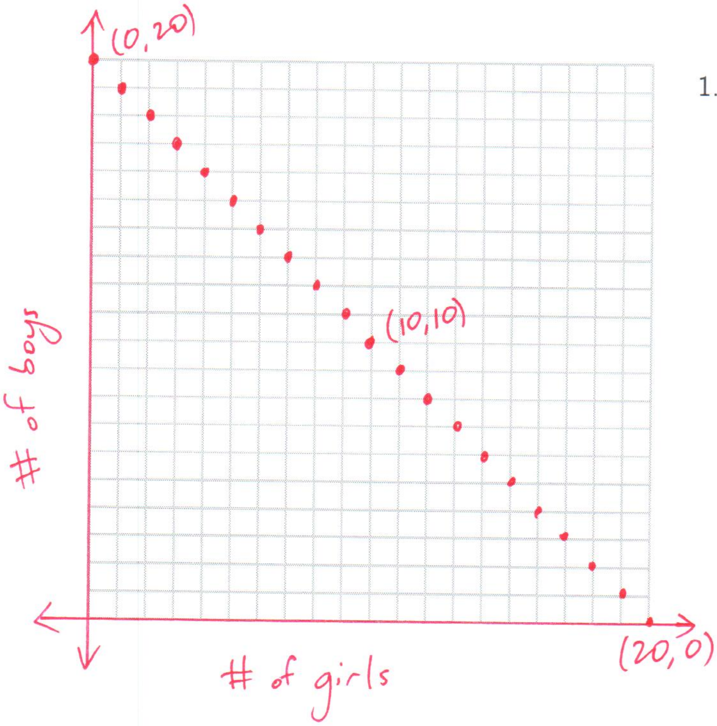
$y = \frac{12}{7}$

$(0, \frac{12}{7})$

Graphical Method

EXPLORATION 2

A math class has 20 students. What are the different possibilities for the number of girls and the number of boys? Let x represent the number of girls and y represent the number of boys.



1. Make a table of the possibilities for x and y. Graph these pairs (x, y) as points on a coordinate system. Note that these points lie on a straight line.

x	y
0	20
1	19
2	18
⋮	⋮
10	10
11	9
⋮	⋮
19	1
20	0

2. What is the equation for this line?

Answers may vary but should be equivalent to $y = -x + 20$
$x + y = 20$, $y = 20 - x$ etc.

3. What is the slope of this line? Explain why this slope makes sense for this problem.

$m = -1$ As the number of girls increases by 1, the number of boys decreases by 1.

4. What are the intercepts for this line? Explain why these two points make sense.

(0, 20) a class of all boys
(20, 0) a class of all girls

5. Are there points on the line that do not represent real possibilities for the number of girls and boys in this case? Explain why.

(10.5, 9.5) is on the line, but we don't have fractional students.
(−2, 22) is on the line, but negative students don't exist.

PROBLEM 4

Terry has total of \$24 to spend at the grocery store on flour and sugar combined. Flour costs \$2 per Kilogram and sugar costs \$3 per Kilogram. Let x represent the amount (in Kilograms) of flour and y represent the amount of sugar (in Kilograms).

1. Write an equation that each possible pair (x, y) must satisfy. Is this the equation for a line? Explain.

   $2x + 3y = 24$ yes, it is an equation in standard form.

2. What is the slope of the line? Why does it make sense?

   $m = \frac{-A}{B} = \frac{-2}{3}$ 2 lbs sugar costs the same as 3 lbs flour. (As one goes up, the other goes down.)

3. What are the intercepts for this line? Explain why these two points make sense.

   x-intercept: $2x + 3(0) = 24$
   $2x = 24$
   $x = 12$ $(12, 0)$ is flour only

   y-intercept: $2(0) + 3y = 24$
   $y = 8$
   $(0, 8)$ is sugar only.

4. For each of the following points, determine if the point is on the line and if the x and y values represent real possibilities for the amount of flour and sugar.
   a. $(9, 2)$
   b. $(\frac{5}{2}, \frac{19}{3})$
   c. $(15, -2)$

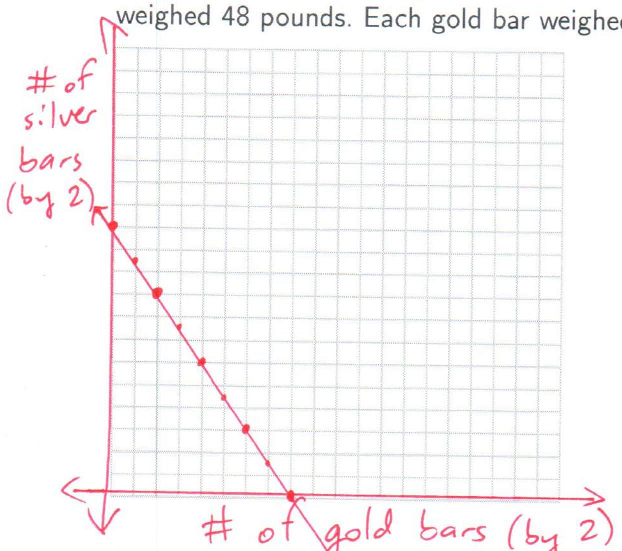
a. $2(9) + 3(2) = 24$
$18 + 6 = 24$ ✓
on the line, possible

b. $2(\frac{5}{2}) + 3(\frac{19}{3}) = 24$
$5 + 19 = 24$ ✓
on the line, only makes sense if sugar and flour can be bought in fractional parts of Kg.

c. $2(15) + 3(-2) = 24$
$30 + (-6) = 24$
$24 = 24$ ✓
on the line, but not possible since sugar is not negative.

EXAMPLE 3

In the past ships carried gold and silver bars. A treasure chest is discovered whose total contents weighed 48 pounds. Each gold bar weighed 3 pounds and each silver bar weighed 2 pounds.



1. Write an equation in standard form describing the possible numbers of gold and silver bars in the chest.

   gold → x, silver → y

   $3x + 2y = 48$

2. Write the equation in slope-intercept form.

   $3x + 2y = 48$

   $\frac{2y}{2} = \frac{-3x + 48}{2}$

   $y = -\frac{3}{2}x + 24$

3. What is the y-intercept and why does it make sense?

   24, because if there are only silver bars, there will be 24 of them, to reach 48 lbs.

4. Graph the line. Are there points on the line that do not represent real possibilities?

   yes, for example $(-2, 27)$ or $(1, \frac{45}{2})$

5. Make a list of the solutions for this equation that represent real possibilities. What patterns do you notice? x is a multiple of 2, y is a multiple of 3

x	0	2	4	6	8	10	12	14	16
y	24	21	18	15	12	9	6	3	0

6. What is the slope? Why does the slope make sense?

   $m = \frac{-3}{2}$

   There are 3 bars of silver added to replace 2 gold bars to keep the weight at 48 lbs, or 2 gold bars added to replace 3 bars of silver.

   (3 bars of silver weighs 6 lbs.
   2 bars of gold weighs 6 lbs.)

SUMMARY (What I learned today)