FRACTIONS 4

Name: Key Date:_____ Period:_____

SECTION 4.4 Addition and Subtraction of Fractions

VOCABULARY

DEFINITION	EXAMPLE
Least Common Denominator: least common multiple of two fractions' denominators	$\frac{1}{3} \to \frac{2}{6}$ and $\frac{1}{2} \to \frac{3}{6}$ (LCM of 2 and 3 is 6.)

Big Idea: How do we add and subtract fractions?

EXPLORATION 1: ADDING AND SUBTRACTING FRACTIONS

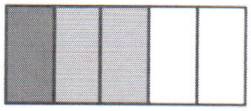
Adding 1 foot to 2 feet equals 3 feet. Combining 1 apple with 2 apples gives 3 apples. In each case, both numbers and units are important. Given these two examples, it seems reasonable to say that the sum of 1 fifth and 2 fifths is 3 fifths. More precisely, in Chapter 2, the linear skip counting model demonstrated that $\frac{3}{5}$ is $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$. Using skip counting, it is easy to see that

$$\frac{2}{5} + \frac{1}{5} = \left(\frac{1}{5} + \frac{1}{5}\right) + \frac{1}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$$

Let's try some more. In the table below, write out the addition problem using unit fractions. Record the sum in simplest form as a proper fraction or mixed number, if possible. The first one is done for you.

Addition Problem	Unit Fractions	Sum in Simplest Form or Mixed Number
$\frac{2}{3} + \frac{2}{3}$	$(\frac{1}{3} + \frac{1}{3}) + (\frac{1}{3} + \frac{1}{3}) = \frac{4}{3}$	$1\frac{1}{3}$
$\frac{3}{5} + \frac{1}{5}$	$(\frac{1}{5} + \frac{1}{5} + \frac{1}{5}) + \frac{1}{5}$	$\frac{4}{5}$
$\frac{5}{8} + \frac{7}{8}$	$(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}) + (\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8})$	$\frac{12}{8} = 1\frac{4}{8} = 1\frac{1}{2}$
$\frac{3}{4} + \frac{3}{4}$	$(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}) + (\frac{1}{4}+\frac{1}{4}+\frac{1}{4})$	$\frac{6}{4} = 1\frac{1}{2}$

How is the sum $\frac{1}{5}+\frac{2}{5}$ computed using the area model? Use a candy bar model. Betsy had $\frac{1}{5}$ of a candy bar, and her friend had $\frac{2}{5}$ of a candy bar like Betsy's.



Together, they have $\frac{3}{5}$ of a candy bar. Express this as $\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$.

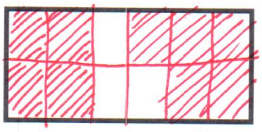
Write rules to generalize the previous discussion of adding fractions.

Answers will vary. When you add n pieces of size k to m pieces of size k, there are n+m pieces of size k. $\frac{a}{d}+\frac{b}{d}=\frac{a+b}{d}$

EXAMPLE 1

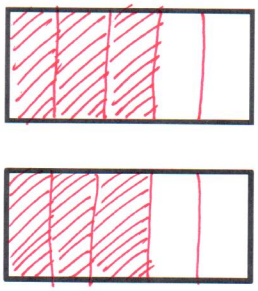
Find the sum and put in simplest form or, if possible, as a mixed fraction. Draw a model to illustrate each.

a. $\frac{4}{12}+\frac{5}{12}$



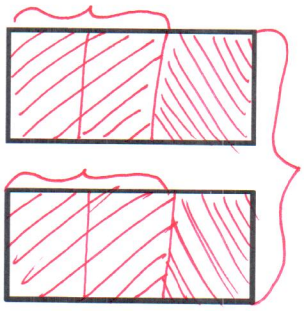
$\frac{9}{12}=\frac{3}{4}$

b. $\frac{3}{5}+\frac{3}{5}$



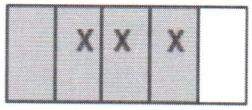
$\frac{6}{5}=1\frac{1}{5}$

c. $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}$



$\frac{6}{3}=2$

The same principle applies when subtracting fractions. $\frac{4}{5}-\frac{3}{5}=\frac{1}{5}$



Find the difference and put in simplest form. Use models to illustrate each. Remember the order of operations.

a. $\frac{5}{6}-\frac{2}{6}$ $= \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}$

b. $\frac{7}{10}-\frac{2}{10}$ $= \frac{7-2}{10} = \frac{5}{10} = \frac{1}{2}$

c. $\frac{5}{8}-\frac{3}{8}-\frac{2}{8}$

$= \frac{5-3}{8} - \frac{2}{8}$

$= \frac{2}{8} - \frac{2}{8} = \frac{0}{8}$

$= 0$

Compute $\frac{7}{9}-\frac{4}{3}$ and explain how to obtain the answer.

Different denominators → must find LCD.

$\frac{7}{9} - \frac{4(3)}{3(3)} = \frac{7}{9} - \frac{12}{9} = \frac{7-12}{9} = \frac{-5}{9}$

EXAMPLE 1

The sum of two fractions with like denominators, $\frac{a}{d}$ and $\frac{b}{d}$, is given by

$$\frac{a}{d}+\frac{b}{d}=\frac{a+b}{d}$$

Write an example to prove the above statement true.

$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$, since "fifths" are similar to "units"

EXAMPLE 2

Describe how to subtract fractions with like denominators. What is the difference, $\frac{m}{n}-\frac{k}{n}$? $\frac{m-k}{n}$

How does your method compare to the addition rule above?

$$\frac{m}{n} - \frac{k}{n} = \frac{m-k}{n}$$

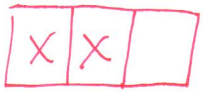
subtracting is like addition of opposites.

$$\frac{m}{n} - \frac{k}{n} = \frac{m}{n} + \left(-\frac{k}{n}\right) = \frac{m+-k}{n} = \frac{m-k}{n}$$

EXAMPLE 3

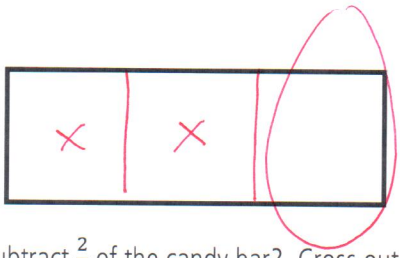
If you eat $\frac{2}{3}$ of a candy bar, how much of the candy bar is left? How can you use subtraction of fractions to answer this question?

$1 - \frac{2}{3} = \frac{3}{3} - \frac{2}{3} : \frac{3-2}{3} = \frac{1}{3}$



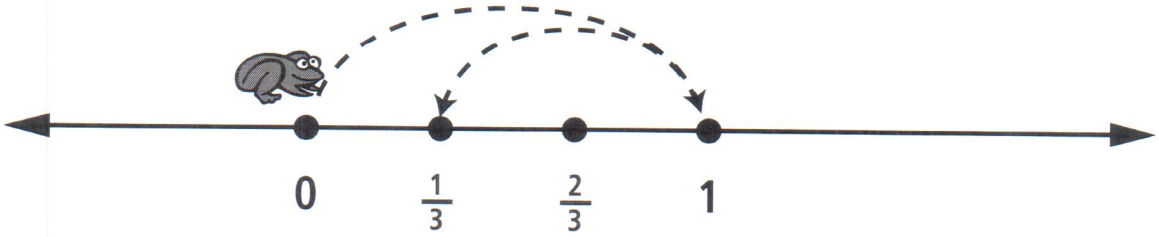
$\frac{1}{3}$ is left

Use the rectangle below to model the candy bar problem.



What happens when you subtract $\frac{2}{3}$ of the candy bar? Cross out the portions that are subtracted to show the difference.

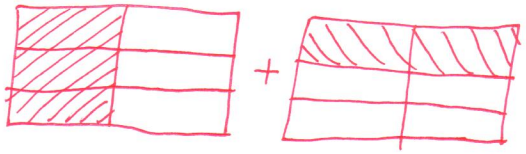
Another way to think of this subtraction is using the frog model. Like the model for subtracting integers, the frog hops 1 unit to the right and then hops backwards a distance of $\frac{2}{3}$ to land on the number $\frac{1}{3}$. This model represents $1 - \frac{2}{3}$.



EXPLORATION 2: UNLIKE FRACTIONS

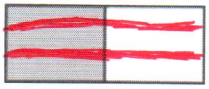
Let's explore how to use the ideas just learned to compute the sum of two fractions when the denominators are not the same.

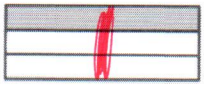
Use the area model to compute $\frac{1}{2} + \frac{1}{3}$.



$= \frac{5}{6}$

Begin by looking at a visual representation.

$\frac{1}{2} =$ 

$\frac{1}{3} =$ 

Is it possible to combine the shaded amounts? In Section 4.2, you discovered that in comparing the fractions $\frac{1}{2}$ and $\frac{1}{3}$, it was helpful to find equivalent fractions for both $\frac{1}{2}$ and $\frac{1}{3}$ to determine which is greater. Modify the picture above to display equivalent divisions of the whole.

Remember, fractional parts must be equal. A third is not the same amount as a half.

To do this, divide the first model horizontally to represent $\frac{1}{2}$ as 3 parts out of 6 parts. Then, divide the second model vertically to represent $\frac{1}{3}$ as 2 parts out of 6 parts. It is easy to see from the model that $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$.

Using the rule for adding fractions with like denominators, the sum is:

$$\begin{array}{r} \frac{1}{2} = \frac{3}{6} \\ +\frac{1}{3} = \frac{2}{6} \\ \hline \frac{5}{6} \end{array}$$

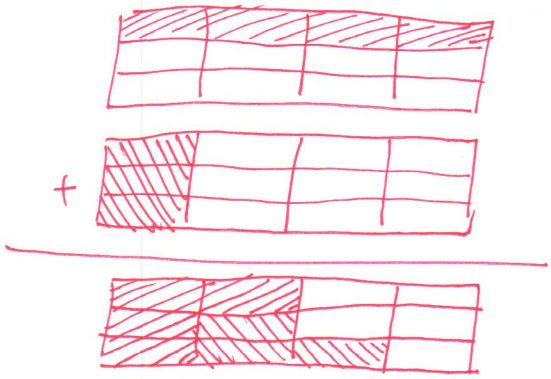
$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{5}{6}$

In order to add the fractions, find common-sized pieces so that the two fractions can be written with the same, or common denominator.

The most important thing to remember when adding fractions is to ensure that you have a common denominator.

EXAMPLE 4

Compute the sum $\frac{1}{3} + \frac{1}{4}$ by first using the area model and then the equivalent fractions property to convert the fractions into equivalent fractions with like denominators.



$\frac{1}{3} = \frac{1(4)}{3(4)} = \frac{4}{12}$

$\frac{1}{4} = \frac{1(3)}{4(3)} = \frac{3}{12}$

$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{4+3}{12} = \frac{7}{12}$

EXAMPLE 5

Find the pattern to add the fractions $\frac{1}{a}$ and $\frac{1}{b}$ and show the process.

$\frac{1}{a} + \frac{1}{b} = \frac{1(b)}{a(b)} + \frac{1(a)}{b(a)} = \frac{b+a}{a \cdot b} = \frac{a+b}{a \cdot b}$

EXPLORATION 3: FINDING COMMON DENOMINATORS

Find three common denominators for the fractions $\frac{1}{6}$ and $\frac{1}{4}$. Write each fraction in equivalent forms using the three denominators. What do you notice about these common denominators? Which denominator would be the best choice for computing the sum $\frac{1}{6} + \frac{1}{4}$? Why?

What three common denominators did you choose? 12, 24, 60

Write 3 equivalent fractions for the unit fractions $\frac{1}{6}$ and $\frac{1}{4}$.

$\frac{1}{6} = \frac{2}{12}, \frac{4}{24}, \frac{10}{60}$

$\frac{1}{4} = \frac{3}{12}, \frac{6}{24}, \frac{15}{60}$

What do you notice about these common denominators?

They are multiples of both 4 and 6.

Which denominator would be the best choice for computing the sum $\frac{1}{6}$ and $\frac{1}{4}$? 12

Explain your reasoning.

It is the smallest denominator, so computations will be easier.

$\frac{2}{12} + \frac{3}{12} = \frac{5}{12}$

EXAMPLE 6

Let's review finding least common multiples. Find the least common multiple of:

a. 3 and 6 6

c. 9 and 15 45

e. 2 and 8 8

b. 12 and 48 48

d. 5 and 7 35

f. 9, 6 and 12 36 $= 3 \cdot 3 \cdot 2 \cdot 2$

$3 \cdot 3$ $3 \cdot 2$ $3 \cdot 2 \cdot 2$

When we add and subtract fractions, having a common denominator is very useful. In order to add $\frac{1}{3} + \frac{1}{6}$, use the equivalent fraction, $\frac{2}{6}$ for $\frac{1}{3}$. The restatement of the problem $\frac{1}{3} + \frac{1}{6}$ to $\frac{2}{6} + \frac{1}{6}$ makes finding the sum of $\frac{3}{6}$ easier to determine.

EXAMPLE 7

For each of the following sums: (1) find a common multiple for both denominators, (2) use it to find equivalent fractions for each fraction, (3) compute their sum and (4) simplify your answer, if necessary.

a. $\frac{1}{9} + \frac{1}{12}$ LCD: 36 b. $\frac{3}{8} + \frac{5}{12}$ LCD: 24 c. $\frac{7}{12} + \frac{5}{18}$ LCD = 36

Hint: It may be easier to rewrite the problems vertically, giving space to write the equivalent fractions with common denominators beside the original fractions. Use the boxes below to show your work. Write all answers in simplest form.

a.

$$\begin{array}{r} \frac{1}{9} \\ +\ \frac{1}{12} \\ \hline \end{array} \qquad \begin{array}{r} \frac{4}{36} \\ +\ \frac{3}{36} \\ \hline \end{array}$$

$$\frac{7}{36}$$

b.

$$\begin{array}{r} \frac{3}{8} \\ +\ \frac{5}{12} \\ \hline \end{array} \qquad \begin{array}{r} \frac{9}{24} \\ +\ \frac{10}{24} \\ \hline \end{array}$$

$$\frac{19}{24}$$

c.

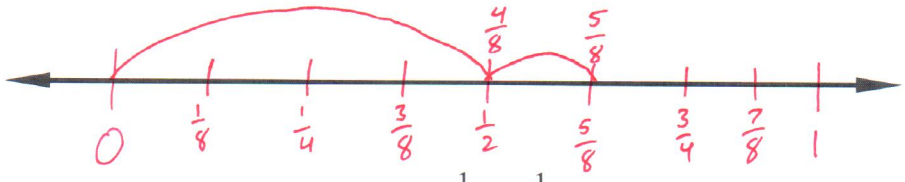
$$\begin{array}{r} \frac{7}{12} \\ +\ \frac{5}{18} \\ \hline \end{array} \qquad \begin{array}{r} \frac{21}{36} \\ +\ \frac{10}{36} \\ \hline \end{array}$$

$$\frac{31}{36}$$

In adding or subtracting fractions, the LCM of the denominators produces the least common denominator or LCD. Using the LCD has the advantage of working with smaller numbers.

EXPLORATION 4: MODELING FRACTION ADDITION

Use the number line below to draw your model for question 1.

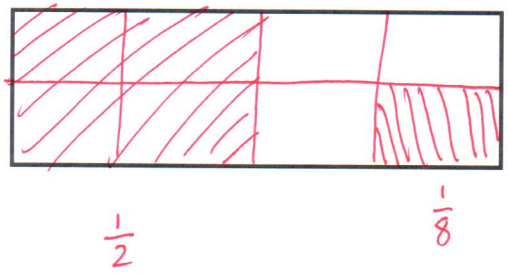


1. Draw a linear model to show the sum of $\frac{1}{2}$ and $\frac{1}{8}$. Explain what denominator you used and why.

$\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$

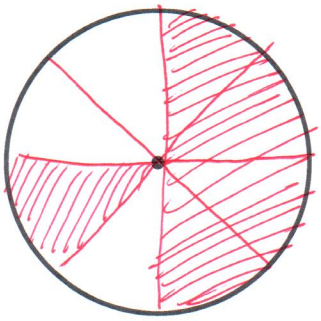
Denominator = 8 because it is the LCM of 2 and 8

2. Use an area model, for example, a rectangle, to show the sum of $\frac{1}{2}$ and $\frac{1}{8}$.



$\frac{5}{8}$

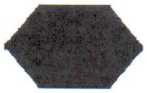
3. Use another area model, for example a circle, to show the sum of $\frac{1}{2}$ and $\frac{1}{8}$.



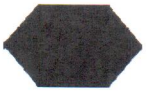
$\frac{5}{8}$

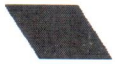
EXPLORATION 5

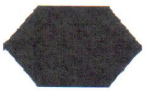
Directions: Look at the diagrams below and see if you can figure out the fractional patterns to answer each question.

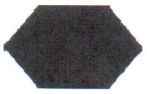
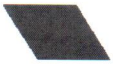
1. If  = 1, then  = $\frac{1}{6}$.

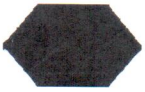
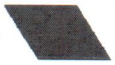
2. If  = 1, then  = $\frac{1}{4}$.

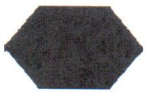
3. If  = 1, then  = $\frac{1}{2}$.

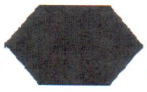
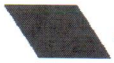
4. If  = 1, then  = $\frac{2}{3}$.

5. If  +  = 1, then  = $\frac{1}{9}$.

6. If  +  = 1, what is  +  ? $\frac{1}{4}$

7. If  +  = 1, what is  +  ? 4

8. If  +  = 1, what is  ? $\frac{3}{7}$

9. If  -  = 1, what is  +  ? 1

PROBLEMS:

1. Raul finished $\frac{3}{5}$ of his homework before dinner. What part must he finish after dinner?

$1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} = \frac{5-3}{5} = \frac{2}{5}$

2. Add or subtract the following fractions:

a. $\frac{5}{a} + \frac{7}{a} = \frac{5+7}{a} = \frac{12}{a}$

b. $\frac{7}{r} - \frac{5}{r} = \frac{7-5}{r} = \frac{2}{r}$

c. $\frac{6}{x} + \frac{5}{y} = \frac{6(y)}{x(y)} + \frac{5(x)}{y(x)} = \frac{6y+5x}{xy}$

3. To make punch for his party, Noah must use $\frac{4}{5}$ liter of lemon-lime soda and $\frac{5}{8}$ liter of pineapple juice. How many liters of punch will Noah make?

$\frac{4}{5} + \frac{5}{8} = \frac{32}{40} + \frac{25}{40} = \frac{32+25}{40} = \frac{57}{40} = \frac{40}{40} + \frac{17}{40} = 1\frac{17}{40}$

LCM = 40 $\quad \frac{4(8)}{5(8)} = \frac{32}{40} \quad \frac{5(5)}{8(5)} = \frac{25}{40}$

4. Emily made a giant sheet cake to share with friends. She gave $\frac{1}{4}$ to Kayla, $\frac{1}{3}$ to Andy, and $\frac{1}{6}$ to Victoria.

How much cake did she give away? $\frac{\frac{1}{4} + \frac{1}{3} + \frac{1}{6}}{\frac{3}{12} + \frac{4}{12} + \frac{2}{12} = \frac{9}{12} = \frac{3}{4}}$

How much cake is left? $1 - \frac{3}{4} = \frac{4}{4} - \frac{3}{4} = \frac{1}{4}$

5. Compute and simplify. Express as a mixed number, if needed.

a. $\frac{1}{4}+\frac{3}{5}+\frac{1}{2}=$ $1\frac{7}{20}$

$\frac{1}{4}=\frac{5}{20}$ $\frac{3}{5}=\frac{12}{20}$ $\frac{1}{2}=\frac{10}{20}$

LCM: 20

$\frac{5}{20}+\frac{12}{20}+\frac{10}{20}=\frac{5+12+10}{20}=\frac{27}{20}=1\frac{7}{20}$

b. $\frac{1}{3}+\frac{5}{6}+\frac{7}{9}=$ $1\frac{17}{18}$

$\frac{1}{3}=\frac{6}{18}$ $\frac{5}{6}=\frac{15}{18}$ $\frac{7}{9}=\frac{14}{18}$

LCM: 18

$\frac{6}{18}+\frac{15}{18}+\frac{14}{18}=\frac{35}{18}=1\frac{17}{18}$

c. $\frac{1}{8}+\frac{1}{3}+\frac{5}{12}=$ $\frac{7}{8}$

$\frac{1}{8}=\frac{3}{24}$ $\frac{1}{3}=\frac{8}{24}$ $\frac{5}{12}=\frac{10}{24}$

LCM: 24

$\frac{3}{24}+\frac{8}{24}+\frac{10}{24}=\frac{21}{24}=\frac{7}{8}$

6. Tell which is greater. Use >, < or =.

a. $\frac{3}{5}+\frac{2}{3}$ $>$ $1-\frac{1}{8}$

$\frac{9}{15}+\frac{10}{15}$ $\frac{8}{8}-\frac{1}{8}$

$\frac{19}{15}>1$ $\frac{7}{8}<1$

b. $\frac{1}{3}+\frac{2}{5}$ $>$ $\frac{7}{9}-\frac{1}{3}$

$\frac{5}{15}+\frac{6}{15}$ $\frac{7}{9}-\frac{3}{9}$

$\frac{11}{15}$ $\frac{7-3}{9}=\frac{4}{9}$

$\frac{11}{15}>\frac{1}{2}$ $\frac{4}{9}<\frac{1}{2}$

c. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ $=$ $2-\frac{3}{5}-\frac{2}{5}$

$\frac{1+1+1}{3}=\frac{3}{3}$ $2-\frac{3}{5}-\frac{2}{5}$

$=1$ $=\frac{10}{5}-\frac{3}{5}-\frac{2}{5}$

$=\frac{10-3-2}{5}$

$=\frac{5}{5}=1$

d. $\frac{2}{5}+\frac{1}{5}$ $>$ $\frac{5}{8}-\frac{1}{4}$

$\frac{2+1}{5}=\frac{3}{5}$ $\frac{5}{8}-\frac{2}{8}$

$=\frac{5-2}{8}$

$=\frac{3}{8}$

Fifths are larger than eighths, so $\frac{3}{5}$ is larger than $\frac{3}{8}$.

SUMMARY (What I learned in this section)