

FRACTIONS

4

Name: Key Date: _____ Period: _____

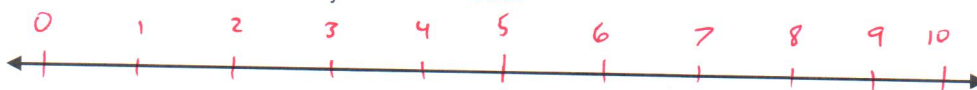
SECTION 4.5 Common Denominators and Mixed Fractions

Big Idea: How do you add and subtract mixed numbers?

EXPLORATION 1: MIXED NUMBERS

Silvia is baking six sheet cakes for a party. The recipe she is using calls for $3\frac{1}{6}$ pounds of refined sugar and $5\frac{1}{4}$ pounds of unrefined sugar. First use the linear model to estimate the total pounds of sugar Silvia needs. Then compute exactly how many pounds of sugar Silvia needs. Explain your process for both the estimation and the exact calculation. Can you use the same process to add other mixed numbers?

Use the number line below to create your linear model.



How much sugar do you estimate Silvia will need? between 8 and 10 pounds

Next, compute exactly how many pounds of sugar Silvia needs.

$$8 + \frac{1}{6} + \frac{1}{4} = 8 + \frac{2}{12} + \frac{3}{12} = 8\frac{5}{12}$$

$$\frac{19}{6} + \frac{21}{4} = \frac{38}{12} + \frac{63}{12} = \frac{101}{12} = 8\frac{5}{12}$$

$$\begin{array}{r} 3\frac{1}{6} \\ + 5\frac{1}{4} \\ \hline 8 + \frac{1}{6} + \frac{1}{4} \end{array} \quad \text{or} \quad \frac{19}{6} + \frac{21}{4}$$

Explain the process you used for the estimation and the calculation.

$3+5$ is 8 and $4+6$ is 10, so $3\frac{1}{6} + 5\frac{1}{4}$ is between 8 and 10 lbs. To calculate exactly, add the whole numbers and then the fractional part of the mixed numbers, or convert to improper fractions, add, and convert back to mixed numbers.

EXAMPLE 1

The following recipe yields about 6 dozen Chocolate Chip cookies.

CHOCOLATE CHIP COOKIES

$2\frac{1}{4}$ cups flour

$\frac{3}{4}$ cups sugar

$\frac{3}{4}$ cups brown sugar

12 oz. chocolate chips

1 tsp. baking soda

$\frac{1}{8}$ tsp. salt

$1\frac{1}{2}$ tsp. vanilla

12 dozen:
multiply by 2.

3 dozen: divide
by 2, or
multiply by $\frac{1}{2}$

How would you adjust the recipe to make 12 dozen cookies? How would you adjust the recipe to make only 3 dozen cookies? Use the table below to organize your calculations.

	6 Dozen	12 Dozen	3 Dozen
flour	$2\frac{1}{4}$ cups	$4\frac{1}{2}$ cups	$1\frac{1}{8}$ cups
sugar	$\frac{3}{4}$ cups	$1\frac{1}{2}$ cups	$\frac{3}{8}$ cups
brown sugar	$\frac{3}{4}$ cups	$1\frac{1}{2}$ cups	$\frac{3}{8}$ cups
chocolate chips	12 oz	24 oz	6 oz
baking soda	1 tsp	2 tsp	$\frac{1}{2}$ tsp
salt	$\frac{1}{8}$ tsp	$\frac{1}{4}$ tsp	$\frac{1}{16}$ tsp
vanilla	$1\frac{1}{2}$ tsp	3 tsp	$\frac{3}{4}$ tsp

EXPLORATION 2

Compute the sum $6\frac{3}{5} + 3\frac{5}{7}$ using three different methods.

1. Improper Fractions:

One approach is to treat this as an ordinary fraction addition problem by converting from mixed to improper fractions and back again. First, convert the mixed fractions to improper fractions:

The whole number 6 is the same thing as 30-fifths. Therefore, $6 + \frac{3}{5} = \frac{33}{5}$. Likewise, $3\frac{5}{7} = \frac{26}{7}$. Now we just need to add the two fractions. Don't forget to rewrite the fractions with common denominators.

$$\begin{array}{r} \frac{33}{5} = \underline{\hspace{2cm}} \\ + \frac{26}{7} = \underline{\hspace{2cm}} \\ \hline \end{array} \quad \frac{33}{5} \cdot \frac{7}{7} = \frac{231}{35} \quad \frac{26}{7} \cdot \frac{5}{5} = \frac{130}{35}$$

$$\frac{231}{35} + \frac{130}{35} = \frac{361}{35} = \frac{350}{35} + \frac{11}{35}$$

$10\frac{11}{35}$

Because you are adding two improper fractions, your answer will also be an improper fraction. Convert your answer to a mixed number.

2. Combining Like Parts:

The improper fractions approach can be cumbersome because it involves working with relatively large numbers. Another approach is to consider each mixed fraction as the sum of an integer and a proper fraction and regroup the whole parts together and the proper fractions together:

$$6\frac{3}{5} + 3\frac{5}{7} = \left(6 + \frac{3}{5}\right) + \left(3 + \frac{5}{7}\right) = (6 + 3) + \left(\frac{3}{5} + \frac{5}{7}\right)$$

This leads to the sum of proper fractions:

$$\begin{aligned} \frac{3}{5} + \frac{5}{7} &= \frac{3 \cdot 7}{5 \cdot 7} + \frac{5 \cdot 5}{7 \cdot 5} \\ &= \frac{21}{35} + \frac{25}{35} \\ &= \frac{46}{35} \\ &= 1\frac{11}{35} \end{aligned}$$

Combining these results, the original sum is:

$$\begin{aligned}
 6\frac{3}{5} + 3\frac{5}{7} &= \left(6 + \frac{3}{5}\right) + \left(3 + \frac{5}{7}\right) \\
 &= (6 + 3) + \left(1 + \frac{11}{35}\right) \\
 &= 10 + \frac{11}{35} \\
 &= 10\frac{11}{35}
 \end{aligned}$$

As you can see, in computing the sum of mixed fractions, it is often easier to separate the mixed fractions as whole parts and fractional parts, add each group and then combine these two partial sums.

3. Vertical Addition:

There is another way to organize and write this same process vertically:

$$\begin{array}{r}
 6\frac{3}{5} \\
 + 3\frac{5}{7} \\
 \hline
 \end{array}
 \longrightarrow
 \begin{array}{r}
 6 \quad \frac{3}{5} \\
 + 3 \quad \frac{5}{7} \\
 \hline
 \end{array}
 \longrightarrow
 \begin{array}{r}
 6 \quad \frac{21}{35} \\
 + 3 \quad \frac{25}{35} \\
 \hline
 9 \quad \frac{46}{35} \\
 \hline
 \end{array}
 \begin{array}{l}
 = 9 + \left(1 + \frac{11}{35}\right) \\
 = 10 + \frac{11}{35} \\
 = 10\frac{11}{35}
 \end{array}$$

How would finding the difference between two mixed fractions be different?

negatives may be an issue, especially if a fraction is subtracted from a smaller fraction

EXAMPLE 2

Compute the following differences:

<p>a. $8\frac{4}{5} - 5\frac{3}{10}$</p> $ \frac{44}{5} - \frac{53}{10} $ $ \frac{88}{10} - \frac{53}{10} = \frac{35}{10} = 3 + \frac{5}{10} $ <p style="text-align: center;">$3\frac{1}{2}$</p>	<p>b. $6\frac{3}{5} - 3\frac{5}{7}$</p> $ (6 - 3) + \left(\frac{3}{5} - \frac{5}{7}\right) $ $ 3 + \left(\frac{21}{35} - \frac{25}{35}\right) $ $ 2 + \frac{35}{35} + \frac{21}{35} - \frac{25}{35} $ $ 2 + \frac{31}{35} = 2\frac{31}{35} $	<p>c. $4 - 2\frac{3}{5}$</p> $ 4 - 2\frac{3}{5} = \frac{20}{5} - \frac{13}{5} = \frac{7}{5} $ <p style="text-align: center;">$1\frac{2}{5}$</p>
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improper fractions

combining like parts

improper fractions

Remember:

- You must have like denominators to add or subtract fractions.
- Writing the problems vertically may help with organization.
- You may use any of the three methods you choose.

EXAMPLE 3

Let's try a few more addition and subtraction problems.

a. $73\frac{8}{9} + 28\frac{1}{3}$

b. $29 - 8\frac{19}{40}$

c. $2\frac{1}{5} - 1\frac{2}{3}$

Use the boxes below to show your work.

<p>a.</p> $73\frac{8}{9} + 28\frac{1}{3}$ $= 73 + 28 + \frac{8}{9} + \frac{3}{9}$ $101 + \frac{11}{9} =$ $101 + 1 + \frac{2}{9} = \mathbf{102\frac{2}{9}}$	<p>b.</p> $20 - 8\frac{19}{40}$ $11 + 8 + 1 - 8 - \frac{19}{40}$ $11 + 1 - \frac{19}{40}$ $11 + \frac{40}{40} - \frac{19}{40} = \mathbf{11\frac{21}{40}}$	<p>c.</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: right;">$2\frac{1}{5}$</td> <td style="text-align: right;">$2\frac{3}{15}$</td> </tr> <tr> <td style="text-align: right;">$- 1\frac{2}{3}$</td> <td style="text-align: right;">$- 1\frac{10}{15}$</td> </tr> <tr> <td colspan="2" style="border-top: 1px solid black; text-align: right;">$1 + \frac{3}{15} - \frac{10}{15}$</td> </tr> <tr> <td colspan="2" style="text-align: right;">$\mathbf{\frac{8}{15}}$</td> </tr> </table> $\frac{15}{15} + \frac{3}{15} - \frac{10}{15} = \mathbf{\frac{8}{15}}$	$2\frac{1}{5}$	$2\frac{3}{15}$	$- 1\frac{2}{3}$	$- 1\frac{10}{15}$	$1 + \frac{3}{15} - \frac{10}{15}$		$\mathbf{\frac{8}{15}}$	
$2\frac{1}{5}$	$2\frac{3}{15}$									
$- 1\frac{2}{3}$	$- 1\frac{10}{15}$									
$1 + \frac{3}{15} - \frac{10}{15}$										
$\mathbf{\frac{8}{15}}$										

combining like terms combining like terms vertical method

PROBLEMS

1. Compute the following sums of mixed fractions using either the horizontal or vertical method. Show all the steps in the process. Simplify your answers if needed.

a. $2\frac{2}{5} + 4\frac{1}{5} =$

b. $4\frac{3}{8} + \frac{5}{12} =$

c. $5\frac{3}{4} + 2\frac{2}{6} =$

Use the boxes below to show your work.

<p>a.</p> $2\frac{2}{5} + 4\frac{1}{5}$ $(2 + 4) + (\frac{2}{5} + \frac{1}{5})$ $6 + \frac{3}{5} = \mathbf{6\frac{3}{5}}$	<p>b.</p> $4\frac{3}{8} + \frac{5}{12}$ $\frac{35}{8} + \frac{5}{12} = \frac{35 \cdot 3}{8 \cdot 3} + \frac{5 \cdot 2}{12 \cdot 2}$ $\frac{105}{24} + \frac{10}{24} = \frac{115}{24}$ $\frac{96 + 19}{24} = \mathbf{4\frac{19}{24}}$	<p>c.</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: right;">$5\frac{3}{4}$</td> <td style="text-align: right;">$5\frac{9}{12}$</td> </tr> <tr> <td style="text-align: right;">$+ 2\frac{2}{6}$</td> <td style="text-align: right;">$+ 2\frac{4}{12}$</td> </tr> <tr> <td colspan="2" style="border-top: 1px solid black; text-align: right;">$7\frac{13}{12}$</td> </tr> <tr> <td colspan="2" style="text-align: right;">$\mathbf{8\frac{1}{12}}$</td> </tr> </table>	$5\frac{3}{4}$	$5\frac{9}{12}$	$+ 2\frac{2}{6}$	$+ 2\frac{4}{12}$	$7\frac{13}{12}$		$\mathbf{8\frac{1}{12}}$	
$5\frac{3}{4}$	$5\frac{9}{12}$									
$+ 2\frac{2}{6}$	$+ 2\frac{4}{12}$									
$7\frac{13}{12}$										
$\mathbf{8\frac{1}{12}}$										

2. Compute the following differences of mixed fractions using either the horizontal or vertical method. Show all the steps in the process. Simplify your answers if needed.

a. $1\frac{3}{14} - 1\frac{1}{7} =$

b. $7\frac{3}{4} - 5\frac{2}{6} =$

c. $9\frac{3}{8} - 4\frac{5}{12} =$

Use the boxes below to show your work.

<p>a.</p> $1 + \frac{3}{14} - \left(1 + \frac{1}{7}\right)$ $1 + \frac{3}{14} - 1 - \frac{2}{14}$ $1 - 1 + \frac{3}{14} - \frac{2}{14} = \frac{1}{14}$	<p>b.</p> $7\frac{3}{4} - 5\frac{2}{6}$ $7\frac{9}{12} - 5\frac{4}{12}$ $\underline{\hspace{1cm}}$ $2\frac{5}{12}$	<p>c.</p> $9 + \frac{3}{8} - 4 - \frac{5}{12}$ $(9-4) + \left(\frac{9}{24} - \frac{10}{24}\right)$ $4 + 1 - \frac{1}{24} = 4 + \frac{24}{24} - \frac{1}{24}$
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3. Kyra has $2\frac{3}{4}$ liters of soda. Jared has $1\frac{5}{8}$ liter of sports drink. How much liquid do they have all together?

$$2\frac{3}{4} + 1\frac{5}{8} = \frac{11}{4} + \frac{13}{8} = \frac{22}{8} + \frac{13}{8} = \frac{35}{8} = \frac{32}{8} + \frac{3}{8} = 4\frac{3}{8} \text{ liters}$$

$$2\frac{3}{4} - 1\frac{5}{8} = \frac{11}{4} - \frac{13}{8} = \frac{22}{8} - \frac{13}{8} = \frac{9}{8}$$

How much more drink does Kyra have than Jared? $1\frac{1}{8}$ liters more

4. On Monday it rained $\frac{3}{4}$ of an inch in Austin, while Dallas received $\frac{5}{6}$ of an inch.

Which city received more rain? Dallas

How much more rain? $\frac{10}{12} - \frac{9}{12} = \frac{1}{12}$ inch

Together, how much rain fell in the cities? $\frac{10}{12} + \frac{9}{12} = \frac{19}{12} = \frac{12}{12} + \frac{7}{12}$

$1\frac{7}{12}$ inches

5. Nicole babysat her younger cousins for $2\frac{1}{4}$ hours on Monday, for $1\frac{1}{2}$ hours on Wednesday and $3\frac{3}{5}$ hours on Friday. How much total time did she spend babysitting her cousins?

$$2\frac{1}{4} + 1\frac{1}{2} + 3\frac{3}{5}$$

LCM of 4, 2, and 5
is 20.

$$2\frac{5}{20} + 1\frac{10}{20} + 3\frac{12}{20}$$

$$\frac{45}{20} + \frac{30}{20} + \frac{72}{20} = \frac{147}{20} = \frac{140}{20} + \frac{7}{20} = 7\frac{7}{20}$$

$7\frac{7}{20}$ hours

6. Sophia's puppy, Boo boo, has 1 cup of food in her dish. If she eats $\frac{4}{7}$ of the food in her dish, how much remains?

$$1 - \frac{4}{7} = \frac{7}{7} - \frac{4}{7} = \frac{3}{7}$$

$\frac{3}{7}$ of the food remains

7. Cameron has $6\frac{1}{3}$ yards of rope. He cuts off $4\frac{3}{8}$ yards of the rope to make a rope swing at the river. How many yards of rope does Cameron have left?

$$6\frac{1}{3} - 4\frac{3}{8}$$

$$(6 - 4) + \left(\frac{1}{3} - \frac{3}{8}\right) = 2 + \left(\frac{8}{24} - \frac{9}{24}\right)$$

$$1 + 1 + \frac{8}{24} - \frac{9}{24} = 1 + \frac{24}{24} + \frac{8}{24} - \frac{9}{24} = 1 + \frac{23}{24}$$

$1\frac{23}{24}$ yards of rope left

SUMMARY (What I learned in this section)
