SECTION 4.2 SUBSTITUTION METHOD

Name: Key **Date:** _____ **Period:** _____

Example

Suppose you were asked to find two numbers that had two conditions:

1. The sum of two numbers is 72
2. One number is 3 times as large as the other number.

We write the equations corresponding to these conditions as

$$x + y = 72$$

and

$$y = 3x.$$

We will now use an algebraic approach called the *substitution method* to solve this system of equations.

Step 1: System of 2 equations with 2 unknowns:
$x+y=72$ and $y=3x$

Step 2: Substitute $3x$ for y in the first equation:
$x+y=72$ becomes $x+3x=72$

Step 3: Simplify and solve for x.

$$x+3x=72$$
$$4x=72$$
$$\frac{4x}{4}=\frac{72}{4}$$
$$x=18$$

Step 4: Substitute $x=18$ into the other equation, $y=3x$:

$$y=3(18)$$
$$y=54.$$

The solution is $(18, 54)$

PROBLEM 1

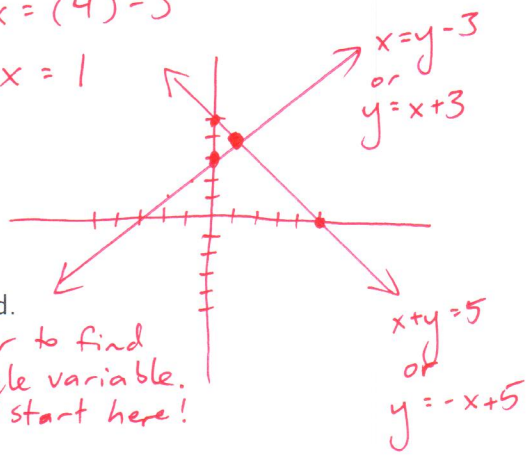
Solve the following system of equations using the substitution method. Write each step carefully and show your work clearly. Check your answer by solving the system graphically.

$$x + y = 5$$
$$x = y - 3$$

$(y-3)+y=5$
$2y-3=5$
$2y-3+3=5+3$
$2y=8$
$\frac{2y}{2}=\frac{8}{2}$ $y=4$

$x=y-3$
$x=(4)-3$
$x=1$

$(1, 4)$



EXAMPLE 1

Solve the following system of equations using the substitution method.

$$4x - y = 15$$
$$-2x + 3y = 12$$

easier to find a single variable. start here!

$4x-y=15$
$4x-y+y-15=15+y-15$
$4x-15=y$

$-2x+3y=12$
$-2x+3(4x-15)=12$
$-2x+12x-45=12$
$10x=12+45$
$10x=57$
$\frac{10x}{10}=\frac{57}{10}$
$x=5.7$

Use $4x-y=15$ (which is $y=4x-15$)
$y=4(5.7)-15$
$y=22.8-15$
$y=7.8$

$(5.7, 7.8)$

Practice Problems

Solve the following systems of equations by substitution.

1. $2x + 3y = 24$
$y = 2x$

$2x+3(2x)=24$
$2x+6x=24$
$8x=24$
$x=3$
$y=2(3)$
$y=6$

$(3, 6)$

2. $3x + 2y = 11$
$y = x - 2$

$3x+2y=11$
$3x+2(x-2)=11$
$3x+2x-4=11$
$5x=15$
$x=3$
$y=x-2$
$y=3-2$
$y=1$

3. $2x - 5y = 24$
$10y = 6x - 48$

$2x-5(.6x-4.8)=24$
$2x-3x+24=24$
$-24 \quad -24$
$-x=0$
$\frac{-x}{-1}=\frac{0}{-1}$
$x=0$

$y=\frac{6}{10}x-\frac{48}{10}$
$y=.6x-4.8$
$y=.6(0)-4.8$
$y=-4.8$

4. $2x - y = 5$
$2y = 4x - 2$

$2x-y=5$
$2x=5+y$
$2x-5=y$

$\frac{2y}{2}=\frac{4x}{2}-\frac{2}{2}$
$y=2x-1$
$2(2x-5)=4x-2$
$4x-10=4x-2$
$-4x \quad -4x$
$-10=-2$

The equations have parallel graphs and no intersection.

PROBLEM 2

In a right triangle, we know that one of the angles measures 90 degrees. The other two angles are acute and must add up to 90 degrees. Suppose the larger acute angle is 26 degrees more than twice the smaller.

1. Sketch the triangle.

larger smaller

2. Write the conditions as a system of equations, using A and B as the measures of the two acute angles. $A + B = 90$ $A = 2B + 26$

3. Use the substitution method to solve the system.

$(A) + B = 90$
$(2B + 26) + B = 90$
$3B + 26 = 90$
$3B = 64$
$B = 21\frac{1}{3}$ (or $\frac{64}{3}$)

$A + B = 90$
$A + (21\frac{1}{3}) = 90$
$A = 90 - 21\frac{1}{3}$
$A = 68\frac{2}{3}$

4. Check your answers to see if the measures add up.

$(21\frac{1}{3}) + (68\frac{2}{3}) = 90$ ✓

$(68\frac{2}{3}) = 2(21\frac{1}{3}) + 26$ ✓

SUMMARY (What I learned today)