SECTION 4.3 METHOD OF ELIMINATION

Name: Key **Date:** ______ **Period:** ______

We now will solve systems of equations using another algebraic approach called the *Elimination Method*.

Example Consider the system:

Equation 1: $5x + y = 25$

Equation 2: $x + y = 13$

1. Multiply both sides of Equation 2 by -1. Label this Equation 3.

$-1(x+y) = (13)(-1)$

Equation 3: $-x - y = -13$

2. Combine Equation 1 and 3 by adding the left hand sides together and setting it equal to the sum of the right hand sides. Label this Equation 4. What happens to the y variable?

$5x + y = 25$
$+ (-x - y = -13)$

Equation 4: $4x = 12$

$(5x+y)+(-x-y) = (25)+(-13)$

$5x - x + y - y = 12$

$4x = 12$

3. Solve for x.

$\frac{4x}{4} = \frac{12}{4}$

$x = 3$

4. Substitute the value of x into Equation 1 or 2 or 3 and solve for y.

Equation 2: $(x)+y = 13$

$(3)+y = 13$

$3+y-3 = 13-3$

$y = 10$

PROBLEM 1

Consider the following systems and discuss which variable you would choose to eliminate in each case and why.

a. $3x + 2y = 10$
$4x - 2y = 4$

y
because 2y and -2y are opposites

c. $x + y = 9$
$-x + 2y = -3$

x because x and -x are opposites

b. $-6x + 5y = 3$
$2x + 5y = 8$

(but one must be negative)
y

d. $4x + y = 17$
$-4x - 5y = -5$

x
because 4x and -4x are opposites

PROBLEM 2

Solve the system, $2x + 2y = 11$ and $x - 2y = -2$ by the elimination method. Check that the solution satisfies both equations simultaneously.

$(2x+2y)+(x-2y)=(11)+(-2)$

$2x+x+2y-2y=11-2$

$3x=9$

$x=3$

$(x)-2y=-2$

$(3)-2y=-2$

$-2y=-5$

$y=\frac{5}{2}$

$y=2\frac{1}{2}$ or 2.5

Check:

$2(3)+2(2.5)\overset{?}{=}11$

$6+5=11$ ✓

$(3)-2(2.5)\overset{?}{=}-2$

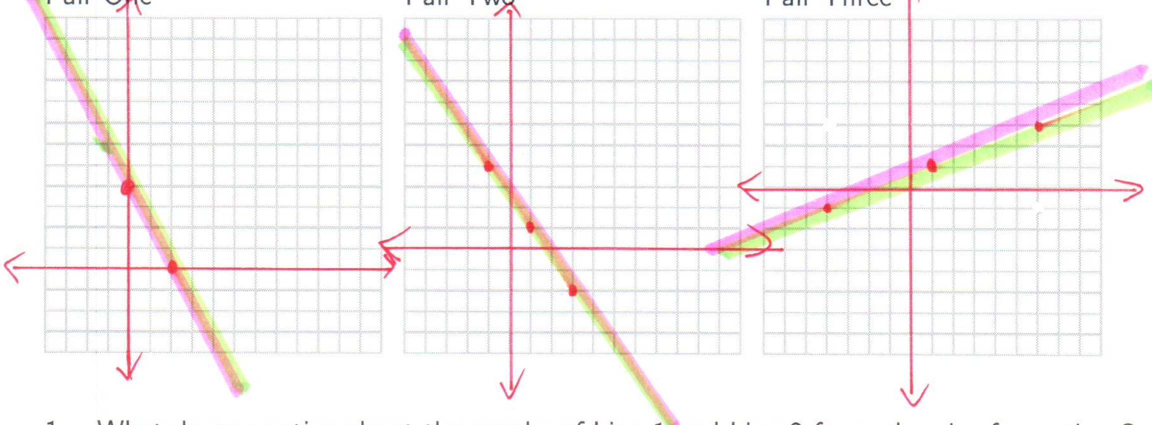
$3-5=-2$ ✓

EXPLORATION 1

What is the effect on the graph of the line when we multiply sides of its equation by a number? In the table below, graph the Line 1 for each initial equation and then graph the Line 2 that is given by the new equation that is derived from the initial equation by multiplying by the indicated factor. Use a new coordinate system for pair of lines.

	Initial Equation	Multiply by	New Equation
Pair One	Line 1: $2x + y = 4$	Factor of 2	Line 2: $4x + 2y = 8$
Pair Two	Line 1: $3x + 2y = 5$	Factor of 3	Line 2: $9x + 6y = 15$
Pair Three	Line 1: $-2x + 5y = 3$	Factor of -2	Line 2: $4x - 10y = -6$

Pair One Pair Two Pair Three



1. What do you notice about the graphs of Line 1 and Line 2 for each pair of equations?

They are the same line.

2. Make a rule about what the effect is on the graph of a line if you multiply its equation by a non-zero factor.

The graph of the line is not affected.

PROBLEM 3

Solve the following system of linear equations by multiplying each equation by an appropriate factor in order to set up the elimination method:

Equation 1: $(-3)(2x + 3y) = (5)(-3)$

Equation 2: $(2)(3x + 5y) = (7)(2)$

$-6x - 9y = -15$

$6x + 10y = 14$

$(-6x - 9y) + (6x + 10y) = (-15) + (14)$

$\boxed{y = -1}$

Eq. 1:

$2x + 3(-1) = 5$

$2x - 3 = 5$

$2x = 8$

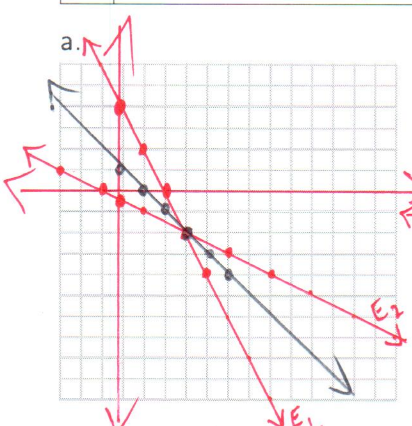
$\boxed{x = 4}$

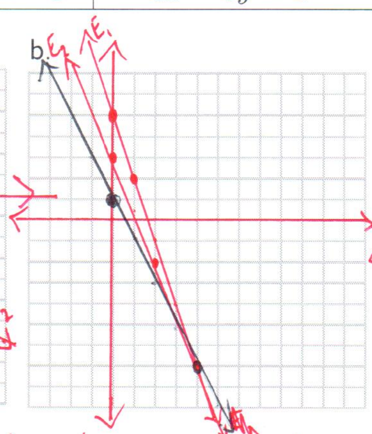
$(4, -1)$

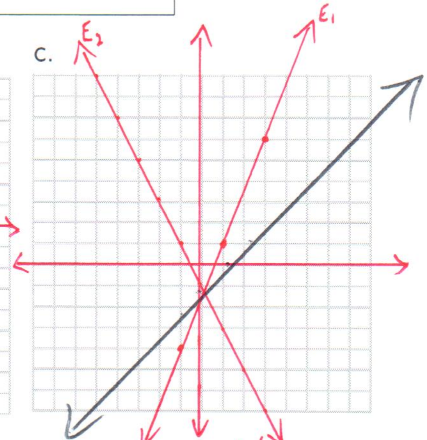
EXPLORATION 2

In the method of elimination, what is the effect on the graphs of two lines if we simply added the corresponding sides to form a new equation that does not eliminate one of the variables? The new equation will still have two variables. How will the line relate to the lines of the initial equations? To explore this question, graph the two lines from each initial system and then add these two equations to form a New Equation. Then graph the line satisfying this New Equation. Use a new coordinate system for linear system of equations.

	Initial Equation	New Equation
a.	Equation 1: $2x + y = 4$ Equation 2: $x + 2y = -1$	Add the equations to get: $3x + 3y = 3$
b.	Equation 1: $3x + y = 5$ Equation 2: $5x + 2y = 6$	Subtract Eq. 1 from Eq. 2 to get: $2x + y = 1$
c.	Equation 1: $5x + -2y = 3$ Equation 2: $2x + y = -1$	Subtract Eq. 2 from Eq. 1 to get: $3x - 3y = 4$

a. 

b. 

c. 

line intersects the two initial equations at the same point

PROBLEM 4

Solve the system of equations by elimination.

$$5x + 4y = -39$$
$$-2\,(3x + 2y) = (-29)(-2) \longrightarrow -6x - 4y = 58$$

$5x + 4y + -6x + -4y = -39 + 58$

$-x = 19$

$x = -19$

$\boxed{(-19, 14)}$

$3x + 2y = -29$
$3(-19) + 2y = -29$
$-57 + 2y = -29$
$2y = -29 + 57$
$2y = 28$
$y = 14$

EXAMPLE 1

Suppose a summer camp orders 4 large pizzas and 2 small pizzas on Monday at a cost of \$57.14 before taxes and 3 large pizzas and 3 small pizzas on Tuesday at a cost of \$52.74 before taxes. What is the cost of each of the large pizzas, assuming they are the same type? What is the cost of each of the small pizzas, assuming they are the same type?

x = cost of a large pizza
y = cost of a small pizza

a large pizza is \$10.99
a small pizza is \$6.59

$4x + 2y = 57.14$ is the cost on Monday

$3x + 3y = 52.74$ is the cost on Tuesday

$-3(4x + 2y) = (57.14)(-3)$ and $4(3x + 3y) = (52.74)(4)$

$-12x - 6y = -171.42$

$12x + 12y = 210.96$

$-12x - 6y + 12x + 12y = -171.42 + 210.96$

$6y = 39.54$

$\boxed{y = 6.59}$

$4x + 2y = 5$

$4x + 2(6.59) = 57.14$

$4x + 13.18 = 57.14$

$4x = 43.96$

$\boxed{x = 10.99}$

SUMMARY (What I learned today)