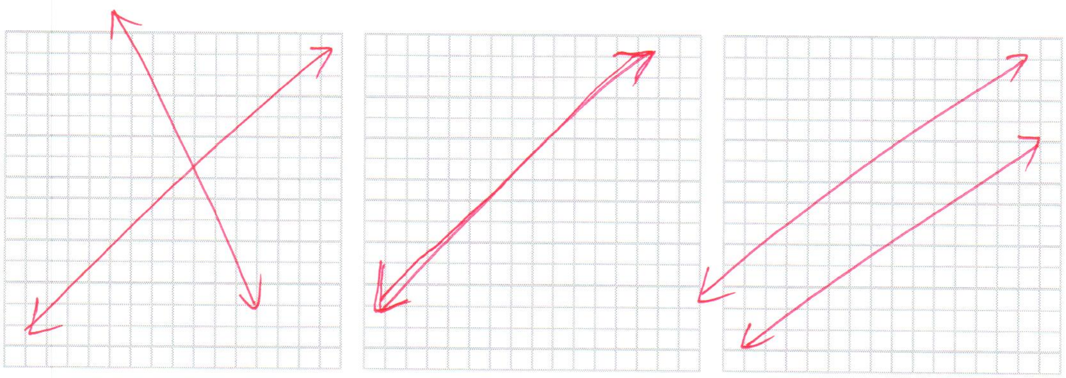
SECTION 4.5 CONSISTENT AND INCONSISTENT SYSTEMS

Name: Key **Date:** ______ **Period:** ______

EXPLORATION 1

Take two uncooked spaghetti noodles and place them on a coordinate grid. Examine how many ways the two noodles do or do not intersect. Classify how many cases can occur. Illustrate the pair(s) in different coordinate grids.



1 intersection

lines are the same

lines are parallel

EXPLORATION 2

1. Write a system with two equations of lines that must be parallel. Give reasons for the equations that you chose. Use the substitution method and proceed to solve the system as in section 4.2. What happened? What does this result suggest? Does this happen anytime you start with two equations of lines that are parallel? Discuss with the class the possible reasons for this.

$y = 7x + 2$

$y = 7x - 3$

They have the same slope.

$7x + 2 = 7x - 3$ (since $y = y$)

$2 = -3$ (subtract $7x$)

This is not true. No values of x and y make both statements true.

2. Write two equivalent but not identical equations that give the same line as their graph. Use substitution to proceed to solve the system. What happened? What does this result suggest? Does this happen anytime you start with two equations that are equivalent. Discuss with the class possible reasons for this.

$x + y = 3 \longrightarrow y = 3 - x$

$2x + 2y = 6$

$2x + 2(3 - x) = 6$

$2x + 6 - 2x = 6$

$6 = 6$

The graphs are the same line, so any (x,y) pair that works for 1 equation makes the other equation true.

EXAMPLE 1

Consider the system: $2x + 3y = 6$ and $2x + 3y = -6$.

1. Explain why the lines given by the equations are parallel.
2. Try to solve the system using the substitution method. What happens? What does this mean?
3. Try to solve the system using the elimination method. What happens? What does this mean?

Why parallel?

They have the same slope, $\frac{-2}{3}$.

Substitution Method:

$2x+3y=6$

$2x=6-3y$

$x=3-\frac{3}{2}y$

$2(x)+3y=-6$

$2(3-\frac{3}{2}y)+3y=-6$

$6-3y+3y=-6$

$6\overset{?}{=}-6$ Not true: no intersection, so the lines are parallel.

Elimination Method:

$-1(2x+3y)=(6)(-1)$

$-2x-3y=-6$

$2y+3y=-6$

$-2x-3y+2x+3y=-6-6$

$0=-12$ Not true: no intersection, so the lines are parallel

Let's see what happens when the system is made of two equivalent equations and use three methods to determine its solutions, if there are any.

EXAMPLE 2

Consider the system $2x + 4y = 2$ and $3x + 6y = 3$.

1. Explain why the lines given by the equations are the same.
2. Try to solve the system using the substitution method. What happens? What does this mean?
3. Try to solve the system using the elimination method. What happens? What does this mean?

Why lines are the same?

$\left(\frac{3}{2}\right)(2x + 4y) = (2)\left(\frac{3}{2}\right)$

$3x + 6y = 3$

OR,

$2x + 4y = 2$

$4y = -2x + 2$

$y = -\frac{1}{2}x + \frac{1}{2}$

$3x + 6y = 3$

$6y = -3x + 3$

$y = -\frac{1}{2}x + \frac{1}{2}$

(same equation)

Substitution Method:

$3x + 6y = 3$

$3x + 6\left(-\frac{1}{2}x + \frac{1}{2}\right) = 3$

$3x + -3x + 3 = 3$

$3 = 3$

Any ordered pair that makes one equation true makes the other equation true.

Elimination Method:

$3(2x + 4y) = (2)3$

$6x + 12y = 6$

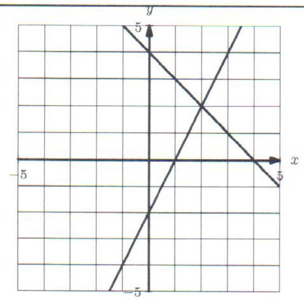
$-2(3x + 6y) = (3)(-2)$

$-6x - 12y = -6$

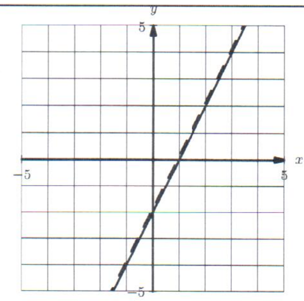
$6x + 12y - 6x - 12y = 6 - 6$

$0 = 0$

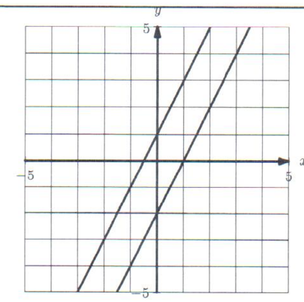
Both x and y are eliminated.



Consistent and **Independent**
$y = 2x - 2$
$y = -x + 4$



Consistent and **Dependent**
$y = 2x - 2$
$2x - y = 2$



Inconsistent
$y = 2x - 2$
$y = 2x + 1$

consistent: system has one or more solutions

inconsistent: system has no solutions

independent: a consistent system with exactly one solution

dependent: a consistent system with more than one solution

SUMMARY (What I learned today)