DECIMAL AND PERCENT REPRESENTATIONS 5

Name: _____Key_____ Date:_____ Period:_____

SECTION 5.2 Operating with Decimals

Big Idea: How do we add, subtract, multiply, and divide decimal numbers?

EXPLORATION 1: ADDITION OF DECIMALS

We'll start this section dealing once again with money. Consider subtracting 3 cents from $35.50.

When you subtract 3¢ from $35.50, you are really subtracting $0.03 from $35.50 to get $35.47. As with addition, it is important to keep in mind the place value and subtract the hundredths from the hundredths, the tenths from the tenths, and so forth. You might have heard the phrase "line up the decimals." This vertical, stacking method assures that the place values also line up to do the calculation.

```
  29.90
   3.49
   1.09
+  0.99
_____
  35.47
```

EXAMPLE 1

Betty is about to take a trip. She fills her car with gas for $29.90 and buys a map for $3.49, a drink for $1.09 and a pack of gum for $0.99. Estimate the cost of her purchase before taxes. Is $40.00 enough to pay for the purchase, excluding tax?   Yes, $30 + 3.50 + 1 + 1 = \$35.50$

Now calculate the exact cost of her purchase before tax. Remember to align the decimals vertically in order to keep the digits in the correct place value.
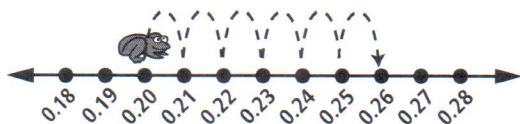
```
  ¹ ² ²
  29.90
   3.49
   1.09
+  0.99
_____
  35.47
```

Betty's actual cost for her purchase will be ____$35.47____.

EXPLORATION 2: LINEAR MODEL

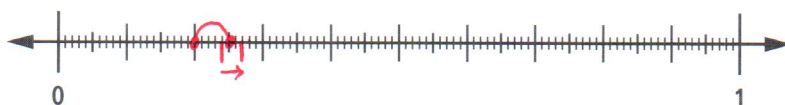You can also use the number line to add decimal numbers.

For example, $0.2 + 0.06 = 0.20 + 0.06$.

Explain how the number line can help to estimate a sum before you calculate the actual total.

You may be able to tell what numbers the sum will be between.

Compute the sums in parts a & b below using the number line. Compute c & d using the stacking method.

a. $0.2 + 0.05 =$ ___0.25___	b. $0.24 + 0.03 =$ ___0.27___
c. $13.7 + 7 =$ ___20.7___ 13.7 $\underline{+7.0}$ 20.7	d. $12.5 + 3.075 =$ ___15.575___ 12.500 $\underline{+3.075}$ 15.575

How do you use the number line to subtract decimals? Compute the following differences using the number line, and then subtract using the traditional stacking method.

a. $0.63 - 0.07$ ___0.56___

$$\begin{array}{r} 0.\overset{5}{\cancel{6}} \\ -\ 0.07 \\ \hline 0.56 \end{array}$$

b. $0.2 - 0.06$ ___0.14___

$$\begin{array}{r} 0.\overset{1}{\cancel{2}}\overset{}{\cancel{0}} \\ -\ 0.06 \\ \hline 0.14 \end{array}$$

EXPLORATION 3: MULTIPLYING DECIMAL NUMBERS

Marie discovers a sale at her favorite store. A shirt she really likes is on sale for $11.59, so Marie decides to buy one shirt in each of her favorite colors: fuchsia, purple, and blue. How much did the 3 shirts cost before tax?

First, let's estimate to see about how much Marie paid. Is $11.59 closer to $11 or ⬭$12⬭?

Use your estimate ☐12 · 3 = ____36____. This amount is your estimate for the cost of the 3 shirts.

Now, let's insert the real price: 11.59 · 3

Step 1: multiply the numbers as if they are integers and ignore the decimal points for now.	1159 x 3 3477
Step 2: Count the number of decimal places in each of your factors and add the decimal place of the factors.	11.59 (2 decimal places) x 3 (0 decimal places) (2 decimal places)

Step 3: Place the decimal point in your answer found in Step 1 by:
 	
  a. starting at the right.
  b. moving left the number of places equal to the sum of the number of decimal places of the factors as found in Step 2.
  c. placing the decimal point in that location.
	11.59
	x 3
	34 ˏ77 (2 decimal places)

Step 4: Check to see if your answer sounds reasonable and close to your estimate above.	yes, $34.77 is close to $36

EXAMPLE 2

Now, let's try multiplying two decimal numbers to see what happens.

Compute 15.22 · 2.3

First, we estimate 15 · 2 = _30_

Now, let's insert the real numbers:

Step 1: It is not necessary to align the decimals. In fact, we can ignore the decimals for now and multiply as usual.	1522 x 23 4566 30440 4566 +30440 35006
Step 2: Now we deal with the decimal. Count the number of decimal places in the factors. **Step 3:** Start at the right of your product in Step 1. Move to the left the sum of the number of places in your factors and place the decimal point at that location.	15.22 (_2_ decimal places) x 2.3 (_1_ decimal places) (_3_ decimal places) 35.006 35.006
Step 4: Check to see if your answer sounds reasonable and close to your estimate above.	yes.

Notice that we do NOT have to line up decimals to multiply.

Try some on your own

a. $8.33 \cdot 5 =$ $\begin{array}{r} 8.33 \\ \times\ \ 5 \\ \hline 41.65 \end{array}$ 2 dec. +0 dec. 2 $\boxed{41.65}$	b. $12.3 \cdot 2.4 =$ $\boxed{29.52}$ $\begin{array}{r} 12.3 \\ \times\ \ 2.4 \\ \hline 492 \\ +2460 \\ \hline 29.52 \end{array}$ 1 dec +1 dec 2	c. $20 \cdot 4.9 =$ $\boxed{98}$ $\begin{array}{r} 20 \\ \times 4.9 \\ \hline 180 \\ +800 \\ \hline 98.0 \end{array}$ 0 dec +1 dec 1
d. $9.075 \cdot 0.2 =$ $\boxed{1.815}$ $\begin{array}{r} 9.075 \\ \times\ \ \ 0.2 \\ \hline 1.8150 \end{array}$ 3 dec +1 dec 4	e. $7.65 \cdot 1.2 =$ $\boxed{9.18}$ $\begin{array}{r} 7.65 \\ \times\ \ 1.2 \\ \hline 1530 \\ +7650 \\ \hline 9.180 \end{array}$ 2 dec +1 dec 3	f. $11.1 \cdot 4.8 =$ $\boxed{53.28}$ $\begin{array}{r} 11.1 \\ \times\ 4.8 \\ \hline 888 \\ +4440 \\ \hline 53.28 \end{array}$ 1 dec +1 dec 2
g. $14.92 \cdot 2.31 =$ $\boxed{34.4652}$ $\begin{array}{r} 14.92 \\ \times\ \ 2.31 \\ \hline 1492 \\ 44760 \\ +298400 \\ \hline 34.4652 \end{array}$ 2 dec +2 dec 4	h. $6.32 \cdot 4.18 =$ $\boxed{26.4176}$ $\begin{array}{r} 6.32 \\ \times 4.18 \\ \hline 5056 \\ 6320 \\ +252800 \\ \hline 26.4176 \end{array}$ 2 dec +2 dec 4	i. $7.42 \cdot 8.6 =$ $\boxed{63.812}$ $\begin{array}{r} 7.42 \\ \times\ \ 8.6 \\ \hline 4452 \\ +59360 \\ \hline 63.812 \end{array}$ 2 dec +1 dec 3
i. $67.22 \cdot 3.456 =$ $\boxed{232.31232}$ $\begin{array}{r} 67.22 \\ \times 3.456 \\ \hline 40332 \\ 336100 \\ 2688800 \\ +20166000 \\ \hline 232.31232 \end{array}$ 2 dec +3 dec 5	j. $5.116 \cdot 54.3 =$ $\boxed{277.7988}$ $\begin{array}{r} 5.116 \\ \times\ \ 54.3 \\ \hline 15348 \\ 204640 \\ +2558000 \\ \hline 277.7988 \end{array}$ 3 dec +1 dec 4	k. $9.644 \cdot 2.2 =$ $\boxed{21.2168}$ $\begin{array}{r} 9.644 \\ \times\ \ \ 2.2 \\ \hline 19288 \\ +192880 \\ \hline 21.2168 \end{array}$ 3 dec +1 dec 4

EXPLORATION 4: DIVIDING DECIMAL NUMBERS

Anastasia bought 15 CDs for a total of $185.85. What is the average cost of each CD?

We recognize this is a division problem because they tell us the <u>total</u> and ask for the average cost of <u>each</u>.

Use the space below to solve $185.85 ÷ 15.

$$15\overline{)185.85}$$

The average cost of 1 CD is ___$12.39___.

$$
\begin{array}{r}
12.39 \\
15\overline{)185.85} \\
-15 \\
\hline
35 \\
-30 \\
\hline
58 \\
-45 \\
\hline
135 \\
-135 \\
\hline
000
\end{array}
$$

EXPLORATION 5: DIVIDING WHEN THE DIVISOR IS GREATER THAN THE DIVIDEND

Vance found that he had $32.75 in his bank. He wants to take it out all in quarters. How many quarters are in $32.75?

To solve this, we have to find out how many times 0.25 will go into $32.75. We set our problem up like this:

<p align="center">"32.75 divided by 0.25"</p>

$$0.25\overline{)32.75}$$

Before we can divide, however, we notice there is a decimal in the divisor. We want to convert 0.25 to a whole number in order to continue. If we multiply 0.25 times 100, it will move the decimal. Try it:

$$
\begin{array}{rl}
100 & \text{0 places} \\
\times\, 0.25 & \underline{\text{2 places}} \\
& \text{2 places}
\end{array}
$$

Now we have a whole number in the divisor. To keep our equation balanced, we must perform the same operation on the dividend.

$$\$32.75 \cdot 100 = \$3{,}275$$

Let's see what we have done to our original problem:

$$0.25 \cdot 100\,\overline{)32.75 \cdot 100} \qquad \text{becomes} \qquad 25\overline{)3275}$$

We now have a whole number in the divisor and we can perform our operation. We find that there are

__131__ quarters in $32.75.

$$
\begin{array}{r}
131 \\
25\overline{)3275} \\
-25 \\
\hline
77 \\
-75 \\
\hline
25 \\
\end{array}
$$

Try some more on your own.

a. $20 \div 2.5 = 200 \div 25$ $$2.5\overline{)20}\qquad \begin{array}{r}008\\25\overline{)200}\\-200\\\hline 0\end{array}$$ $\boxed{8}$	b. $40.25 \div 0.5$ $$\begin{array}{r}80.5\\5\overline{)402.5}\\-40\\\hline 025\end{array}$$ $\boxed{80.5}$	c. $0.36 \div .08$ $$\begin{array}{r}4.5\\8\overline{)36.0}\\-32\\\hline 40\\-40\\\hline 0\end{array}$$ $\boxed{4.5}$
d. $0.72 \div 0.08$ $$\begin{array}{r}09\\8\overline{)72}\\-72\\\hline 0\end{array}$$ $\boxed{9}$	e. $124 \div 0.12$ 124.00000 $$\begin{array}{r}1033.\overline{3}\\12\overline{)12400}\\-12\\\hline 040\\-36\\\hline 40\\-36\end{array}$$ $\boxed{1033.\overline{3}}$	f. $52 \div 1.25$ $\boxed{41.6}$ $$\begin{array}{r}0041.6\\125\overline{)5200.0}\\-500\\\hline 200\\-125\\\hline 750\\-750\\\hline 0\end{array}$$
g. $210 \div 4.2$ $$\begin{array}{r}50\\42\overline{)2100}\\-210\\\hline 00\end{array}$$ $\boxed{50}$	h. $180 \div 2.4$ $$\begin{array}{r}75\\24\overline{)1800}\\-168\\\hline 120\\120\\\hline 0\end{array}$$ $\boxed{75}$	i. $180 \div 0.24$ $$\begin{array}{r}750\\24\overline{)18000}\\-168\\\hline 120\\-120\\\hline 00\end{array}$$ $\boxed{750}$
j. $450 \div 1.8$ $$\begin{array}{r}250\\18\overline{)4500}\\-36\\\hline 90\\-90\\\hline 0\end{array}$$ $\boxed{250}$	k. $197.03 \div 1.75$ $$\begin{array}{r}112.588\\175\overline{)19703.000}\\-175\\\hline 220\\-175\\\hline 453\\-350\\\hline 1030\\-875\\\hline 1550\\-1400\\\hline 1500\\-1400\\\hline 100\end{array}$$	l. $0.324 \div 3.6$ $$\begin{array}{r}0.09\\36\overline{)3.24}\\-324\\\hline 000\end{array}$$ $\boxed{0.09}$

k rounded to the hundredths
place = $\boxed{112.59}$

PROBLEMS

1. What must you <u>always</u> do before adding or subtracting decimal numbers?

   line up the decimals

2. Compute the following:

a. 0.67 + 0.54	b. 0.93 + 9.12	c. 27.13 + 1.9
$\begin{array}{r} 0.67 \\ + 0.54 \\ \hline 1.21 \end{array}$	$\begin{array}{r} 0.93 \\ + 9.12 \\ \hline 10.05 \end{array}$	$\begin{array}{r} 27.13 \\ + 1.90 \\ \hline 29.03 \end{array}$
d. 0.75 − 0.24	e. 7.602 − 3.12	f. 89.42 − 4.63
$\begin{array}{r} 0.75 \\ - 0.24 \\ \hline 0.51 \end{array}$	$\begin{array}{r} 7.602 \\ - 3.120 \\ \hline 4.482 \end{array}$	$\begin{array}{r} 89.42 \\ - 4.63 \\ \hline 84.79 \end{array}$
g. 0.665 + 3.45 + 2.40	h. 56.43 + 24.110 + 1.24	i. 67.34 + 2.359 + 0.911
$\begin{array}{r} 0.665 \\ 3.450 \\ + 2.400 \\ \hline 6.515 \end{array}$	$\begin{array}{r} 56.430 \\ 24.110 \\ + 1.240 \\ \hline 81.780 \end{array}$	$\begin{array}{r} 67.340 \\ 2.359 \\ + 0.911 \\ \hline 70.610 \end{array}$

3. Determine which of the following pairs of numbers is closer together. Explain your answer.

   a. 0.4 and 0.5   OR   (0.41 and 0.45)

   b. 0.79 and 0.81   OR   (0.792 and 0.801)

   a. $0.5 - 0.4 = \boxed{0.1}$ b. $0.81 - 0.79 = \boxed{0.02}$

   $0.45 - 0.41 = \boxed{0.04}$ $0.801 - 0.792 = \boxed{0.009}$

   $0.04 < 0.1$ $0.009 < 0.02$

4. Alexandria has $423.79 in her savings account. She wants to buy a shirt for $17.99 and jeans on sale for $68 including tax. If she buys these clothes, how much will be left in her savings account?

   $\begin{array}{r} 423.79 \\ - 17.99 \\ \hline 405.80 \end{array}$ $\begin{array}{r} 405.80 \\ - 68.00 \\ \hline 337.80 \end{array}$ $337.80

5. When you multiply decimal numbers, is it necessary to line up the decimals? __no__. Explain.

The number of decimal places matters, but they can be put in at the end.

6. Find the product of 1.5y, if y is equal to the values given below.

a. y = 0.3	b. y = 10	c. y = 0.09
1.5 1 dec × .3 +1 dec .45 2 **0.45**	1.5 ×10 15.0 **15**	4 1.5 ×0.09 .135 **0.135**
d. y = 12.7 12.7 × 1.5 **19.05** 635 +1270 1905	e. y = 15.114 15.114 × 1.5 **22.6710** 75570 +151140 22.6710 **= 22.671**	f. y = 0.007 0.007 × 1.5 **0.0105** 35 + 70 000.0105

7. Gil has 153.75 fluid ounces of hydrogen peroxide to pour evenly into 15 flasks. How many ounces will he pour into each flask?

```
     1 0.25
15 | 153.75
    -15
     037
     -30
      75
     -75
       0
```

10.25 fl. oz.

8. Parker has 29.75 fl. oz. of fluid to pour into flasks. Each flask must have 1.75 fl. oz. How many flasks can he fill?

29.75 ÷ 1.75

2975 ÷ 175

```
        1 7
175 | 2975
     -175
     1225
    -1225
        0
```

17 flasks

SUMMARY (What I learned in this section)

_____

_____

_____

_____

_____

_____