DECIMAL AND PERCENT REPRESENTATIONS 5

Name: Key Date:_______ Period:______

SECTION 5.3 Numbers as Decimals and Fractions

VOCABULARY

DEFINITION	EXAMPLE
Equivalent: different ways to write the same quantity	$\frac{1}{2}$ and 0.5
Fraction: The fraction $\frac{m}{n}$ is the quotient of $m \div n$ (where n is not 0)	$\frac{1}{7}$, $\frac{8}{15}$, $\frac{1}{2}$, $\frac{3}{4}$

Big Idea: How can we represent the same quantity in decimal and fraction forms?

In the past, you have probably referred to one-half of a dollar as $0.50 or 50 cents. One half is a fraction that is equal to 0.50, a decimal. We say that $\frac{1}{2}$ is **equivalent** to 0.50. In this section, we will review how a fraction can be represented as a decimal number and how some decimals can be represented as fractions.

EXPLORATION 1: DECIMAL NUMBERS AND THEIR NAMES

Let's begin this exploration by recreating the place value chart. Write the names of the place values in the chart below, as shown in the example below.

	THOUSANDS	Hundreds	Tens	ONES	•	TENTHS	Hundredths	Thousandths
a			5	0	.	0	0	5
b				2	.	0	9	
c	4			6	.	2	5	
d			1	6	.	1	6	
e		3	2	0	.	7		

Now, read the following words and write the decimal form in your place value chart.

a) Fifty and five thousandths

b) Two and nine hundredths

c) Four thousands six and twenty-five hundredths

d) Sixteen and sixteen thousandths

e) Three hundred twenty and seven tenths

Decimals can be read in terms of the place values that the digits occupy. For example, 0.47 is read "forty seven one hundredths." You know that the fractional representation $\frac{47}{100}$ is also read "forty seven one hundredths." You can also locate the numbers on the number line as the same point. The decimal form 0.47 and the fractional form $\frac{47}{100}$ actually represent the same value.

In converting a decimal to a fraction, we take advantage of the fact that we use the base ten system to write each decimal number. For example, the number 0.3 is called three tenths and so is equivalent to $\frac{3}{10}$. The number 0.35 is read as 35 hundredths and so is the same as $\frac{35}{100}$.

EXPLORATION 2: DECIMALS TO FRACTIONS

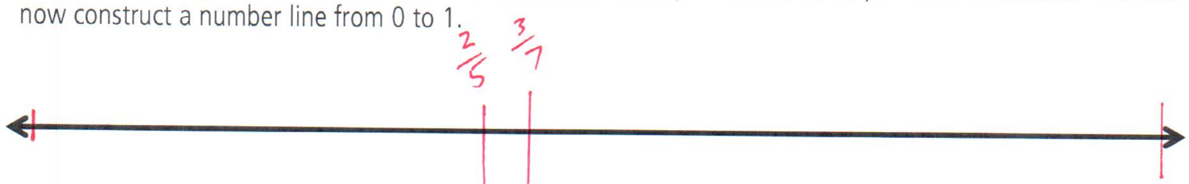
Does the fraction $\frac{1}{5}$ have a decimal form? If we buy 5 bananas for \$1, we know that each banana costs \$1 ÷ 5 = \$0.20 or 20 cents. In other words, each banana costs $\frac{1}{5}$ dollar because it takes 5 (\$0.20) to make a whole dollar. So 1 ÷ 5 = 0.20 or 20 hundredths. But the decimal 0.20, or twenty hundredths, has the same name as the fraction $\frac{20}{100}$. Does this mean the fraction $\frac{1}{5}$ is equal to $\frac{20}{100}$? These are equivalent fractions, so the decimal 0.20 and the fractions $\frac{1}{5}$ and $\frac{20}{100}$ all represent the same quantity and are equal.

Write the fractional form of the following decimal numbers. It may help to first say the word correctly using place value. Remember to simplify all fractions, if possible.

a) 0.7 $\frac{7}{10}$	b) 0.01 $\frac{1}{100}$	c) 0.216 $\frac{216}{1000} = \frac{108}{500} = \frac{54}{250} = \boxed{\frac{27}{125}}$
d) 0.903 $\boxed{\frac{903}{1000}}$	e) 5.4 $5\frac{4}{10} = \boxed{5\frac{2}{5}}$	f) 4.7 $4\frac{7}{10}$

EXPLORATION 3: LINEAR MODEL FOR FRACTIONS

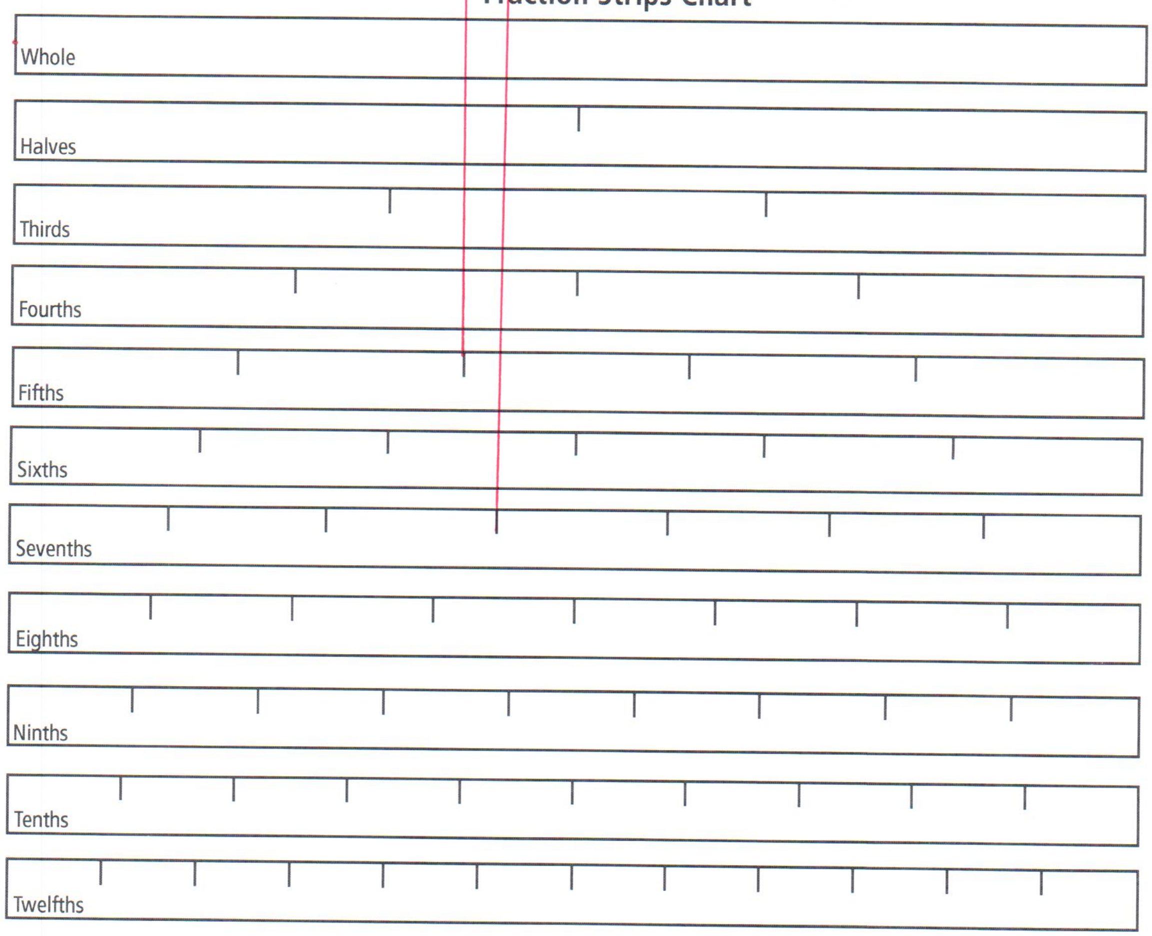
On a number line, each integer corresponds to a point. Recall that there are many other points between each pair of integers on the number line, and each of these points also corresponds to a number. We will now construct a number line from 0 to 1.



Use the number line above as your line master. The rows of the Fraction Strips Chart below represent the result of folding the whole into equal parts to represent fractions as we did in the Linear Model Activity. The computer that created it allows the folds to be very accurate. For example, using the Fraction Strips Chart, determine which fraction is greater: $\frac{2}{5}$ or $\frac{3}{7}$.

$\frac{3}{7}$ is greater

Fraction Strips Chart



EXAMPLE 1

Convert the following fractions to decimal form. You may refer to your linear model as needed.

a. $\frac{1}{4} =$ 0.25
$\frac{2}{4} =$ 0.5
$\frac{3}{4} =$ 0.75

b. $\frac{1}{5} =$ 0.2
$\frac{2}{5} =$ 0.4
$\frac{3}{5} =$ 0.6
$\frac{4}{5} =$ 0.8

c. $\frac{1}{8} =$ 0.125
$\frac{2}{8} =$ 0.25
$\frac{3}{8} =$ 0.375
$\frac{5}{8} =$ 0.625

d. $\frac{1}{3} =$ $0.\overline{3}$
$\frac{1}{30} =$ $0.0\overline{3}$
$\frac{1}{9} =$ $0.\overline{1}$
$\frac{1}{90} =$ $0.0\overline{1}$

EXAMPLE 2

We see that the fraction bar $\frac{m}{n}$ is just another symbol used for division, m ÷ n. To convert a fraction to a decimal, it helps to remember that a fraction is a division problem. $\frac{1}{4}$ is the same thing as 1 ÷ 4.

Consider the next example involving money.

If you buy four apples for a dollar, how much does each apple cost? Dividing \$1 by 4 yields, \$1÷4 = \$0.25, or 0.25 dollars. We can also say each apple costs a quarter or $\frac{1}{4}$ of a dollar. So $\frac{1}{4}$ and 0.25 are equal or equivalent. Now we ask, "What decimal is equivalent to $\frac{1}{3}$?" We could also ask what is $\frac{1}{3}$ of a dollar? We can use our new rule to see that $\frac{1}{3}$ is equivalent to the quotient of 1 ÷ 3. The quotient is a repeating decimal, 0.33333... which can be written as $0.\overline{3}$. The bar over the 3 tells us that the digit 3 repeats without end. Previously, we discovered that this quotient is $1 \div 3 = 0.3333\ldots = 0.\overline{3}$. While $\frac{1}{3}$ of a dollar is $\$0.\overline{33}$, we cannot practically divide \$1 into 3 equal parts with our present set of coins, so we often approximate to $\$0.\overline{33}$.

There are other fractions that equal repeating decimals:

$2 \div 3 = 0.6666\ldots = 0.66\overline{6} = 0.\overline{6}$

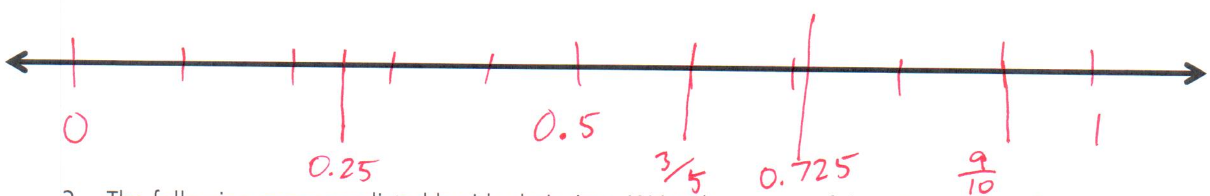
$1 \div 6 = 0.1666\ldots = 0.166\overline{6} = 0.1\overline{6}$

PROBLEMS

1. Complete the missing parts in the table below.

Fractions	Decimals	Words
$\frac{5}{10} = \frac{1}{2}$	0.5	Five tenths
$\frac{35}{100} = \frac{7}{20}$	0.35	Thirty-five hundredths
$\frac{3}{4}$	0.75	Seventy-five hundredths
$6\frac{3}{100}$	6.03	Six and three one hundredths
$8\frac{3}{12} = 8\frac{1}{4}$	8.25	Eight and twenty-five hundredths

2. Order the following numbers from least to greatest and plot them on the number line below: $0.25, \frac{9}{10}, 0.725, \frac{3}{5}$



3. The following racers are listed beside their time. Write the names of the winners from first to last place. (Remember the fastest person has the smallest time.)

Mimi	7.324 minutes	1st:	Eric
Stu	7.3 minutes	2nd:	Tess
Chandra	7.1001 minutes	3rd:	Chandra
Tess	6.2 minutes	4th:	Stu
Eric	6.11 minutes	5th:	MiMi

4. Determine whether a decimal or fraction representation is more appropriate in the following situation.

   a. Leila is baking a cake. $2\frac{1}{2}$ cups of sugar or 2.5 cups of sugar? fraction

b. Patricia is putting gas in her car: $12\frac{3}{5}$ gallons or 12.6 gallons? decimal

c. You are saving money: a $12.50 deposit or $12\frac{1}{2}$ deposit? decimal

5. Convert each decimal to an equivalent fraction or mixed number. Simplify if needed.

a. 0.8 $\frac{8}{10}$ $\boxed{\frac{4}{5}}$	b. 0.07 $\frac{7}{100}$	c. 4.019 $\boxed{4\frac{19}{1000}}$ or $\frac{4019}{1000}$
d. 0.38 $\frac{38}{100} = \boxed{\frac{19}{50}}$	e. 1.55 $1\frac{55}{100} = \boxed{1\frac{11}{20}}$ or $\frac{31}{20}$	f. 3.005 $3\frac{5}{1000} = \boxed{3\frac{1}{200}}$ or $\frac{601}{200}$
g. 0.84 $\frac{84}{100} = \boxed{\frac{21}{25}}$	h. 50.37 $\boxed{50\frac{37}{100}}$ or $\frac{5037}{100}$	i. 2.12 $2\frac{12}{100} = \boxed{2\frac{3}{25}}$ or $\frac{53}{25}$

6. Taylor measured the width of the spine of her Math Explorations textbook and found it to measure between 2.5 and 2.7 cm. Name 3 possible measurements for the width of the spine: Answers will vary

2.6 cm, 2.51 cm, 2.63284 cm

7. Justin's sandcastle measured $8\frac{1}{4}$ feet tall while Kliftan's measured $7\frac{3}{5}$ feet tall. What is the difference in height between these two amazing sandcastles? 0.65 feet

$8\frac{1}{4} - 7\frac{3}{5}$

$8.25 - 7.6$

$$\begin{array}{r} 8.25 \\ -\ 7.60 \\ \hline 0.65 \end{array}$$

SUMMARY (What I learned this section)