DECIMAL AND PERCENT REPRESENTATIONS 5

Name: Key Date:_____ Period:_____

SECTION 5.4 Fractions, Decimals, and Percent

VOCABULARY

DEFINITION	EXAMPLE
Percent: out of a hundred	70% is $\frac{70}{100}$

Big Idea: How do we represent quantities in fractional, decimal, and percent form?

EXPLORATION 1: FRIENDLY FRACTIONS

You have learned that fractions such as $\frac{3}{4}$ can be written as the equivalent fraction $\frac{75}{100}$. This equivalent fraction can also be represented by the decimal 0.75. In some instances, this number can then be converted to 75 percent, 75%. The word percent means "out of a hundred" in Latin.

Decimals can be converted to fractions by reading the decimal form. For example 0.75 is read "seventy-five one hundredths" which in fractional form is 75/100. This in turn says 75 out of 100 or 75%. Notice how three different forms, the decimal, fractional, and percent are all referring to the same quantity.

Similarly, you can reverse the pattern of converting percentages to decimals by dividing the percent by 100.

For example 75% is equivalent to $75 \div 100 = \frac{75}{100} = 0.75$. Even if the percent includes a decimal part, simply divide by 100 to get its decimal equivalent. For example, 6.48% is equivalent to $6.48 \div 100 = 0.0648$.

FRACTION	DECIMAL	PERCENT
$\frac{3}{4}$	0.75	(0.75) (100)= 75%
$\frac{12}{25}$	0.48	(0.48) (100)= 48%
$\frac{7}{10}$	0.7	(0.7)(100) = 70%
$\frac{3}{15}$	0.2	(0.2)(100) = 20%
$\frac{6}{1000}$	0.006	(0.006)(100) = 0.6%
$\frac{9}{20}$	0.45	(0.45)(100) = 45%

While the process used in the table is useful for many fractions, it only works for "friendly fractions." A friendly fraction is one that can easily be converted to fraction form with denominators that are tenths, hundredths, and thousandths.

How do you convert a fraction like $\frac{15}{20}$ into a decimal and a percent?

Since the denominator 20 can easily convert to 100, let's write an equivalent fraction with 100 as the denominator.

$\frac{15}{20} = \frac{5 \cdot 15}{5 \cdot 20} = \frac{75}{100}$

Once you know the hundredths, convert to decimal form. To convert to a percent, multiply the decimal value times 100.

$\frac{15}{20} = \frac{?}{100} =$ 0.75 (Decimal) = 75% (percent)

EXAMPLE 1

A class of 25 students has 12 girls. What percent of the class are girls?

Fraction -----> Decimal -----> Percent

$\frac{12}{25} = \frac{48}{100}$, 0.48, 48%

EXAMPLE 2

In the following table, all denominators easily convert to 10, 100, or 1,000. Use the same strategy as in Example 2 to complete the table.

Fraction	Decimal	Percent
$\frac{4}{5} = \frac{80}{100}$	0.8	80%
$\frac{28}{50} = \frac{56}{100}$	0.56	56%
$\frac{3}{4} = \frac{75}{100}$	0.75	75%
$\frac{1}{5} = \frac{20}{100}$	0.2	20%
$\frac{9}{10} = \frac{90}{100}$	0.9	90%
$\frac{18}{20} = \frac{90}{100}$	0.9	90%

EXPLORATION 2: MORE FRACTIONS TO PERCENTS

In the previous exploration, the fractions easily converted to a denominator found in the place value chart, making our job much easier.

Sometimes, however, there is more work to be done. Consider the fraction $\frac{5}{8}$. The denominator, 8, does not have 10 or 100 as a multiple.

Remembering that a fraction is a division problem, we can still make the conversion.

$$\frac{\text{Numerator}}{\text{Denominator}} \quad \frac{5}{8} \quad \frac{\text{Dividend}}{\text{Divisor}}$$

The fraction five-eighths can be written as 5÷8, which we compute below:

$8\overline{)5}$	**Step 1:** Place the dividend inside the division "house", and the divisor outside. This shows we are dividing 5 by 8.
$8\overline{)5.0}$	**Step 2:** Insert a decimal and a zero at the end of the dividend.
$\begin{array}{r} 0. \\ 8\overline{)5.0} \end{array}$	**Step 3:** Begin the process of division. 8 will go into 5 zero times. Write the 0 in the quotient and bring the decimal straight up.
$\begin{array}{r} 0.6 \\ 8\overline{)5.0} \\ \underline{4.8} \\ .20 \end{array}$	**Step 4:** Continue to divide. 8 goes into 50 six times with a remainder of 2.
$\begin{array}{r} 0.625 \\ 8\overline{)5.0} \\ \underline{4.8} \\ 0.20 \\ \underline{0.16} \\ 0.40 \\ \underline{0.40} \\ 0 \end{array}$	**Step 5:** Continue to add zeroes to the dividend to see if the decimal terminates.
$0.625 \cdot 100\% = 62.5\%$	**Step 6:** $\frac{5}{8}$ is equivalent to 0.625 or 62.5%

Now it is your turn! Convert the fraction $\frac{18}{21}$ to a percent.

$\frac{18}{21} = \frac{6}{7}$

$\frac{18}{21} \approx 0.857 = 85.7\%$

$$\begin{array}{r} 0.857 \\ 7\overline{)6.000} \\ -56 \\ \hline 40 \\ -35 \\ \hline 50 \\ 49 \\ \hline 1 \end{array}$$

Hopefully you noticed that the fraction is not in simplest form. Must you simplify the fraction before turning it into a percent? Write your thoughts below: No, but it will be more work.

Remember, a fraction is just a division problem's quotient.

Use the space below to convert the fraction to a percent. Write your percent answer on the line below.

$\frac{18}{21} \approx 0.857 = 85.7\%$

$$\begin{array}{r} 0.857 \\ 21\overline{)18.000} \\ -168 \\ \hline 120 \\ -105 \\ \hline 150 \\ -147 \\ \hline 3 \end{array}$$

EXAMPLE 3

As you remember, some decimals do not terminate. For example, $\frac{1}{3}$ is a repeating decimal because:

$1 \div 3 = 0.333\ldots$

To convert a repeating decimal to a percent, we still multiply by 100. Since the decimal never ends, however, we must truncate, or simply cut the decimal off, before multiplying. In the example below, the decimal was only written to the thousandths place.

0.333	3 places
X 100	+ 0 places
33.300	3 places

We see that 0.333 will convert to 33.3%. Knowing that $\frac{1}{3}$ is really 0.333… then, $\frac{1}{3} = 33.\overline{3}\,\%$. Notice the repeating bar is written over the tenths place.

EXAMPLE 4

Complete the table below by converting the fractions with division. Use a repeating bar, if needed.

Fraction	Decimal	Percent
$\frac{5}{6}$	$0.8\overline{3}$	$83.\overline{3}\%$
$\frac{1}{8}$	0.125	12.5%
$\frac{2}{3}$	$0.\overline{6}$	$66.\overline{6}\%$
$\frac{1}{9}$	$0.\overline{1}$	$11.\overline{1}\%$

$6\overline{)5.000}$ → 0.83, -48, 20, -18, 20, ⋮

$3\overline{)2.00}$ → $.66$, -18, 20, -18, 2, ⋱

$8\overline{)1.000}$ → 0.125, -8, 20, -16, 40, -40, 0

$9\overline{)1.00}$ → 0.11, -9, 10, -9, 1, ⋱

EXPLORATION 3: PERCENT TO FRACTION

We have converted a lot of fractions to percentages. But, what about changing a percent to a fraction? Recall the meaning of the word "percent".

Percent means out of a hundred.

Therefore, 69% means "69 out of 100", or $\frac{69}{100}$.

Likewise, 149% = $\frac{149}{100}$, an improper fraction. We could convert this to a mixed number, $1\frac{49}{100}$.

What would 350% look like? $3\frac{50}{100}$ or $3\frac{1}{2}$

Now let's try converting a percent to a fraction and a decimal:

79% means 79 out of a hundred, or as a fraction $\frac{79}{100}$.

We have converted the percent to a fraction. Now, use place value to convert to a decimal. Complete the statement below:

79% = $\frac{79}{100}$ (fraction) = .79 (decimal)

EXAMPLE 5

In a small bag of 32 pieces of mixed candy, there are 4 pieces of lemon candy.

What percent of the candy is lemon? $\frac{4}{32} = \frac{1}{8}$

$$8\overline{)1.000} = 0.125$$

-8, 20, -16, 40, -40, 0

12.5 % is lemon.

What percent of the bag is not lemon?

$32 - 4 = 28$ $\quad \frac{28}{32} = \frac{7}{8}$

$$8\overline{)7.000} = 0.875$$

-64, 60, -56, 40, -40, 0

OR:

$100 - 12.5 = 87.5$

87.5 % is not lemon.

PROBLEMS

1. Complete the following table. Write all fractions in simplest form.

Fraction	Decimal	Percent
$\frac{18}{24} = \frac{3}{4}$	0.75	75%
$\frac{75}{100} = \frac{3}{4}$	0.75	75%
$\frac{86}{100} = \frac{43}{50}$	0.86	86%
$\frac{157}{100} = 1\frac{57}{100}$	1.57	157%
$\frac{4}{5}$	0.8	80%
$\frac{90}{100} = \frac{9}{10}$	0.9	90%
$\frac{98}{1000} = \frac{49}{500}$	0.098	9.8%
$\frac{5}{1000} = \frac{1}{200}$	0.005	0.5%
$\frac{4}{6} = \frac{2}{3}$	$0.6\overline{6}$	$66.\overline{6}\%$
$\frac{19}{100}$	0.19	19%
$\frac{22}{1000} = \frac{11}{500}$	0.022	2.2%

Total = 48

boys:

$$12\overline{)5.0000} = 0.4166$$

-48, 20, -12, 80, -72, 80, -72, 8, ⋮

girls:

$$12\overline{)7.000} = 0.583$$

-60, 100, -96, 40, -36, 4, ⋮

2. There are 28 girls and 20 boys in a math competition.

   What percent of the competition are boys? $\frac{20}{48} = \frac{5}{12} = 41.\overline{6}\%$

   What percent of the competition are girls? $\frac{28}{48} = \frac{7}{12} = 58.\overline{3}\%$

OR:

$100 - 41.\overline{6} = 58.\overline{3}$

3. Brittany is planning a slumber party for her youth group. She took a survey of food choices for dinner. 8 kids chose tacos, 16 chose pizza, 6 chose hamburgers, and 2 could not decide. What percent of the group chose each food:

Total: $8+16+6+2=32$

25 % chose tacos $\frac{8}{32}=\frac{1}{4}$

50 % chose pizza $\frac{16}{32}=\frac{1}{2}$

18.75 % chose hamburgers $\frac{6}{32}=\frac{3}{16}$

6.25 % could not decide $\frac{2}{32}=\frac{1}{16}$

$16\overline{)3.0000}=0.1875$ (−16, 140, −128, 120, −112, 80, −80, 0)

$16\overline{)1.0000}=0.0625$ (−96, 40, −32, 80, −80, 0)

4. Out of 200 parents who voted, only 84 parents voted in favor of school uniforms.

a. What percent of parents were in favor of school uniforms? 42%

$\frac{84}{200}=\frac{42}{100}$

b. What percent of parents were not in favor of school uniforms? 58%

$200-84=116$ $\frac{116}{200}=\frac{58}{100}$

5. Mr. Garcia has 30 students in his fifth period class. Five-eighths of the students turned in an extra credit assignment.

What percent of students did not turn in the extra credit assignment? 37.5%

$1-\frac{5}{8}=\frac{8}{8}-\frac{5}{8}=\frac{3}{8}$

$8\overline{)3.000}=0.375$ (−24, 60, −56, 40)

6. Compare the following numbers using <, >, or =.

a. $\frac{3}{4}$ [=] 0.75

b. $\frac{29}{200}=\frac{14.5}{100}$ [<] 32%

c. $\frac{7}{8}$ [<] 0.9 (0.875)

d. $\frac{43}{50}=\frac{86}{100}$ [>] 40%

e. $\frac{1}{3}$ [>] .33 ($0.333\ldots$)

f. 1.3 [>] 100%

g. $\frac{4}{5}$ [<] 85% (0.80)

h. 0.9 [>] 82.5%

7. In Mrs. Cason's Theater Arts Class there are 12 sixth graders, 7 seventh graders, and 6 eighth graders. Of these students, 8 sixth graders, 4 seventh graders, and 5 eighth graders are in the school talent show. Compute the percent of the theater arts students in each grade that will participate in the talent show.

6th $\frac{8}{12} = \frac{2}{3} = 66.6\%$

7th $\frac{4}{7} \approx 57.14\%$

8th $\frac{5}{6} = 83.\overline{3}\%$

$$7\overline{)4.0000} = .5714 \qquad 6\overline{)5.00} = .83$$

-35, 50, -49, 10, -7, 30, -28, 2 ...

-48, 20, -18, 2 ...

Which grade has the highest percent of participation? 8th

Which grade has the lowest percent of participation? 7th

SUMMARY (What I learned in this section)