EQUATIONS, INEQUALITIES, AND FUNCTIONS 6

Name: Key Date:_____ Period:_____

SECTION 6.1 Patterns and Sequences

VOCABULARY

DEFINITION	EXAMPLE
Pattern: a list that changes in a predictable way	2, 4, 6, 8, 10, ...
Term: a number in a list or pattern.	The second term is 4 in the above pattern.
Sequence: a number list	1, 3, 5, 7, ...
Constant Difference: terms increasing by the same amount	the constant difference above is 2.
Arithmetic Sequences: a list with a constant difference (sequence)	1, 4, 7, 10, ... (constant difference is 3)

Big Idea: How do we find and describe patterns in sequences of numbers?

EXPLORATION 1: PATTERNS AND SEQUENCES

A list of numbers can have interesting characteristics. For example, what pattern do you notice in the following list of numbers?

1, 3, 5, 7, 9, 11, 13, 15,...

Write down a few of your observations about the numbers below:

- All are odd.
- Add two to get the next term.
- Double the term number and subtract 1 to get the term.

The number list, or sequence, below has 8 terms listed. What could the next 4 terms be?

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, ...

How did you determine the 9th through 12th terms that you wrote above? Explain.

Add 2.

Position	Term
1	3
2	6
3	9
4	12
5	15
6	18

Think about this: If we wanted to know what the twenty-seventh term would be, how could we do that without skip counting each term through the twenty-seventh? Write your ideas below.

Odd numbers: 2(term number) − 1

3(term number)

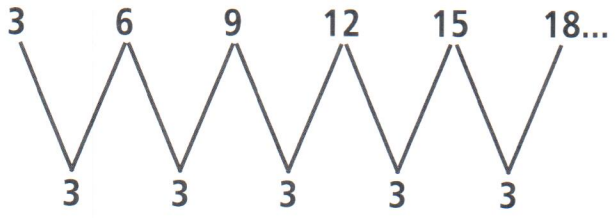
Now, let's try using a table to aide us in our discovery. Consider the sequence

3, 6, 9, 12, 15, 18...

We would say the first term is 3, the second term is 6, and so on. Let's make a table with "Position" or "Term number" and "Term" as the heading.

see above.

Hint: We can draw in large Vs to show that the term is changing by the same amount with each change of position.



One observation is that the terms are increasing by 3. We say there is a **constant difference**, 3, between the adjacent terms. Lists of numbers with a constant difference are called **arithmetic sequences**. Are any of the other sequences in the exploration above arithmetic sequence? Identify the constant differences.

Yes, odds have a constant difference of 2.

Another observation that you may have noticed is "horizontally". Is there a pattern that relates the position of the number with the number in the list? The pattern should relate the term number 1 to the term 3, the term number 2 to the term 6, the term number 3 to the term 9, the term number 4 to the term 12 and so on.

We can describe the pattern between the position, n, and the corresponding term as $3n$. Check to see if this expression for the n^{th} term works. The 20th term should be $3 \cdot 20 = 60$.

Write a statement explaining how we could find the n^{th} term.

Plug in n and solve. The 40th term is 3(40) = 120.

Let's try exploring some more sequences using tables.

EXAMPLE 1

Predict the next 3 terms, and then write an expression using the term number to find the n^{th} term.

a. 6, 12, 18, 24,...

Position	Term	
1	6	
2	12	6
3	18	6
4	24	6
5	30	6
6	36	6
7	42	6

b. 4, 7, 10, 13,...

Position	Term	
1	4	
2	7	3
3	10	3
4	13	3
5	16	3
6	19	3
7	22	3

c. 2, 5, 8, 11,...

Position	Term	
1	2	
2	5	3
3	8	3
4	11	3
5	14	3
6	17	3
7	20	3

Expression (a): $6n$ **Expression (b):** $3n+1$ **Expression (c):** $3n-1$

d. 2, 6, 10, 14, 18, 22,...

Position	Term	
1	2	
2	6	→ 4
3	10	→ 4
4	14	→ 4
5	18	→ 4
6	22	→ 4
7	26	→ 4
8	30	→ 4
9	34	

Expression (d): $4n - 2$

e. 2, 6, 18, 54, 162,...

Position	Term	
1	2	
2	6	→ 4
3	18	→ 12
4	54	→ 36
5	162	→ 108
6	486	→ 324
7	1458	→ 972
8	4374	→ 2916

Expression (e): $2 \cdot 3^{n-1}$

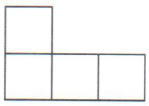
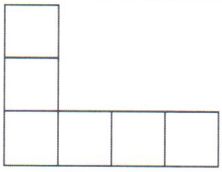
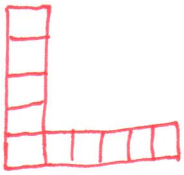
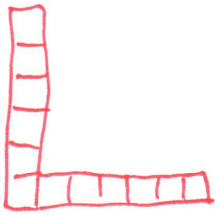
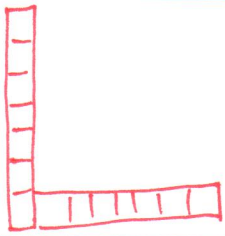
f. 7, 12, 17, 22, 27,...

Position	Term	
1	7	
2	12	→ 5
3	17	→ 5
4	22	→ 5
5	27	→ 5
6	32	→ 5
7	37	→ 5
8	42	

Expression (f): $5n + 2$

EXPLORATION 2: BLOCK PATTERNS

Some patterns may involve shapes that change in a predictable manner. The example below involves shapes created by blocks. A pattern is observable both as a shape and as a number sequence.

Position	Figure	Term: Number of Blocks
1		4
2		6
3		8
4		10
5		12
6		14

If the pattern continues, determine how many blocks would be in the 4th figure; in the 5th figure; in the 6th figure. What pattern do you observe? Write an expression for the number of blocks in the n^{th} figure.

$2n+2$ (or $2(n+1)$)

PROBLEMS

Use tables to describe the number sequences below. Include the next three terms in the sequence. Write an expression for what you think will be the n^{th} term.

1. 10, 100, 1000, 10000, ... 10^n
2. 7, 12, 17, 22, 27, 32, ... $5n+2$
3. 3, 7, 11, 15, ... $4n-1$

1.

position	term
1	10
2	100
3	1,000
4	10,000
5	100,000
6	1,000,000
7	10,000,000

2.

position	term
1	7
2	12
3	17
4	22
5	27
6	32
7	37
8	42
9	47
10	52

3.

position	term
1	3
2	7
3	11
4	15
5	19
6	23
7	27

4. Find the next 3 missing terms, then write the expression for each sequence:

a. 4, 10, 16, 22, 28, 34, 40, ... (6 6 6)

Expression: $2n-2$

b. 4, 9, 14, 19, 24, 29, 34, ... (5 5 5)

Expression: $5n-1$

5. Madeline planted a sunflower seed and took measurements each week to chart its growth. The seedling measured 4 inches the first week, 6 inches the second week, and 8 inches the third week. If this pattern continues, how tall will the sunflower be the fourth week? 10 If this pattern continues, what expression could we use to find the measure of the n^{th} week? $2n+2$

4, 6, 8, 10

6. Julio puts his allowance in the bank each month. He had $75 in January, $150 in February, and $225 in March. If he continues this pattern of saving, how much money will his account hold in June? $450 in December? $900

1	2	3	4	5	6	7	8	9	10	11	12
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
75	150	225	300	375	450						

75 75

$75n$ $75(12) = 900$

7. A kindergartner is putting the cars of a toy train in a color pattern: red, blue, red, blue… Assuming this pattern continues, what color will the twentieth car be? blue

1 2 3 4
red blue red blue

even is blue
odd is red

8. Determine if the following sequence is an arithmetic sequence or non- arithmetic sequence. Explain how you know.

3 5 7 9
1, 4, 9, 16, 25, …

non-arithmetic. There is not a constant difference.

9. Create a table to find the next 5 terms in the following sequence. Write the expression you could use to find the n^{th} term:

12, 22, 32, 42, 52, 62, 72, 82, 92

10 10 10

10n + 2

SUMMARY (What I learned in this section)