Name: Key Date:_______ Period:______

SECTION 6.2 EQUATIONS

VOCABULARY

DEFINITION	EXAMPLE
Equation: a math sentence with an equality sign (=) that relates 2 expressions	$3+2=m-4$
Balanced Equation: 2 expressions that are equal, but change in the same way and remain equal	$3+2+4=m-4+4$
Solve an Equation: find the value that makes an equation true	$9=m$
Equivalent Equations: equations with the same solution	$5=m-4$ $5+4=m-4+4$
Subtraction Property of Equality: subtract the same amount to get an equivalent equation | if $A=B$ then $A-C=B-C$	$2=x+4$ $2-4=x+4-4$ $-2=x$

Big Idea: How do you translate a word problem into an equation?

EXPLORATION 1: WRITING EQUATIONS

Let's begin with the sentence "A number is 3 more than 7." You could figure out what this number is with relative ease, but how can you write this mathematically? You may recall that you can use numbers, variables and operations to form expressions. We can now combine these expressions to form a mathematical sentence called an equation. An **equation** is a math sentence with an equality sign, =, that relates two expressions.

For example, "three more than fifteen" is an expression. It includes numbers and an operation (addition), but it does not include an equality sign. If we state that "three more than fifteen *is* eighteen" then we have changed our expression to an equation. The word "is" indicates equality. Let's look at an example.

EXAMPLE 1

Translate the sentence "A number is 24 more than 17" into an equation.

SOLUTION

Step 1: We give the unknown number a name, N, and write "$N =$ the number." N is a variable. It represents the number we are trying to find.

Step 2: We translate the sentence into an equation.

A Number	is	24 more than 17
N	$=$	$17 + 24$

So the equation form of the sentence is $N = 17 + 24$.

Since $17 + 24 = 41$, we can conclude that $N = 41$. N now represents a known quantity, 41, instead of an unknown quantity. Therefore, we say that we have **solved** the equation for N.

Let's try using the steps above to translate other sentences into equations. Let the variable x represent a number.

a. x less than 22 equals 19 $22 - x = 19$

b. 4 less than x equals 27 $x - 4 = 27$

c. 6 more than x is 107 $x + 6 = 107$

d. x plus 12 is 30 $x + 12 = 30$

Now, let's look at an example using the number line.

EXAMPLE 2

What number is twice as large as six?

Use the number line to also illustrate the solution.

SOLUTION

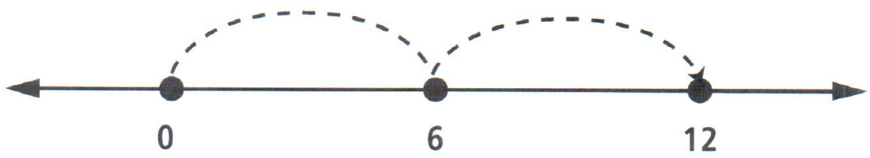
Step 1: We define a variable to represent our unknown number. Let T be a number that is twice as large as six.

Step 2: The statement "A number is twice as large as six" translates as $T = 2 \times 6 = 2 \cdot 6 = (2)(6)$.

When we put the symbols 2 and 6 next to each other with parentheses around each, it is understood that we mean to multiply them. The small dot is also a symbol for multiplication. So $(2)(6) = 2 \cdot 6 = 2 \times 6$. We usually do not use the symbol $\times$, however, since it could be confused with a variable x.

Step 3: We know $2 \cdot 6 = 12$, so $T = 12$.

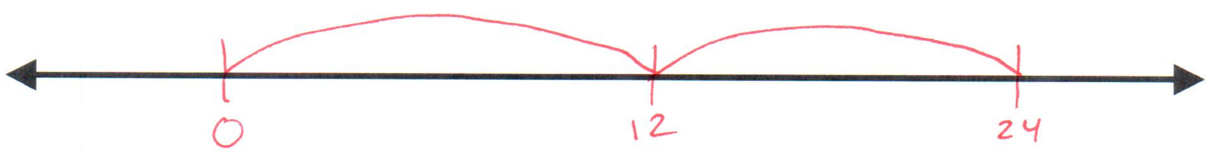
Step 4: Check. Is 12 a number that is twice as large as 6? Yes.



Now it's your turn. Use the line below to create a number line model the for the next word problem. Write an equation to find the solution.

Pat purchased a dozen doughnuts. The number of milk cartons Terry purchased is twice the number of doughnuts Pat purchased. How many milk cartons did Terry purchase? M = number of milk cartons

$M = 2(12)$



What equation did you write to solve the problem? $M = 2 \cdot 12$

How many milk cartons did Terry purchase? 24

EXAMPLE 3

Translate the sentence "A number is 2 less than four times 10" into an equation and solve for the unknown variable. Does your answer make sense?

$N = 40 - 2$

$N = 38$

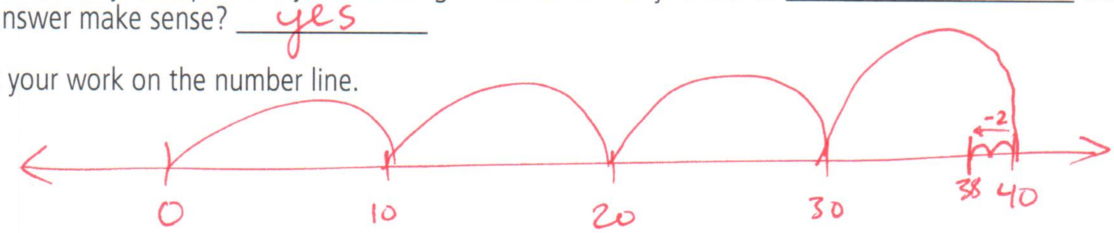
Step 1: Select a variable to represent your unknown value. Write it: N

Step 2: Now write an equation for the statement in EXAMPLE 3: $N = 4 \cdot 10 - 2$

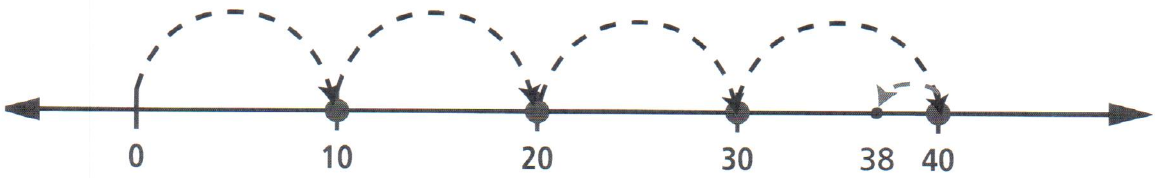
Step 3: The left side of your equation equals 38 and the right side equals 38. Is your equation balanced?

Step 4: Check your equation by substituting in the value that you found: $38 = 4 \cdot 10 - 2$ Does your answer make sense? yes

Check your work on the number line.



Notice on the number line below that 4 times 10 is 40 and 2 less than 40 is 38.



EXPLORATION 2: CHARTING THE PROCESS

We have seen how we can use numbers and variables to translate problems into equations. Consider the problem, "Jeremy is 9 years old. In how many years will Jeremy be 15 years old?"

How might you begin this problem? Did you define a variable? If so, how did you use this variable?

Here is a step-by-step approach. Do your steps resemble the following?

Step 1: Define your variable

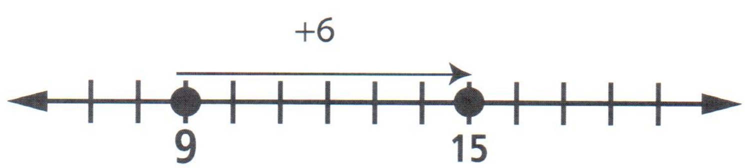
Y = the number of years it takes for Jeremy to reach 15.

Step 2: Translate the problem into an equation

We know that 15 is Y more than 9, so we write $15 = 9 + Y$, an equation with one variable, Y.

Step 3: Solve for the unknown variable

If you look on the number line, you'll notice you have to move right 6 units to go from 9 to 15. So $Y = 6$.



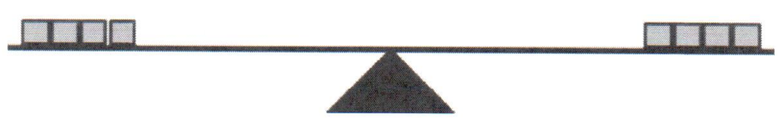
Step 4: Check your answer

Substitute $Y = 6$ into the original equation to see that $15 = 9 + 6$.

Using the 4 step process discussed above, create a poster showing how the steps should be used to solve a problem. Begin by writing the problem and continue by showing how each step helps you solve the problem.

Another way to visualize an equation is with a balance scale.

This is a balance scale:



When we put a weight on one side of the scale, we must place the same weight on the other side in order for the scale to be balanced. If a scale is balanced and equal weights are added or subtracted from both sides of the scale, the scale will remain balanced.

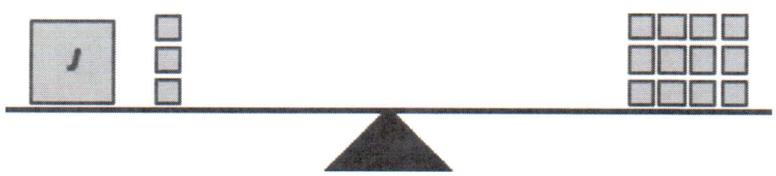
In much the same way, an equation is a statement that two expressions are equal. We can think of the expressions on each side of the equality sign as representing the weight placed on each side of a balanced scale. When we add or subtract the same amount from each side of the equation, the equation will remain **balanced**.

EXAMPLE 4

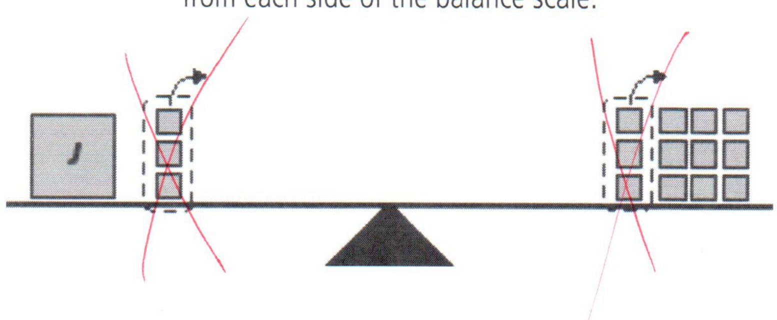
If Jeremy were three years older, he would be the same age as his twelve-year-old sister. What is Jeremy's age? We let J be Jeremy's age, and translate the sentence into an equation as follows:

Jeremy's age	three years older	same age as	twelve-year-old sister
J	$+3$	$=$	12

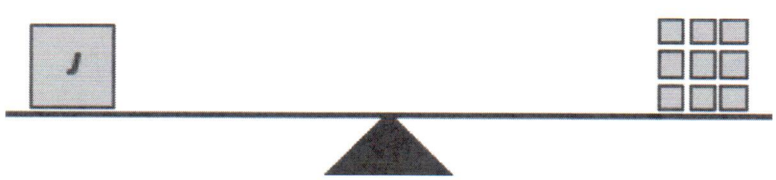
Now we have the equation $J + 3 = 12$. The unknown is J, Jeremy's age. Pictorially, this sentence says that $J + 3$ is the same as 12, which we can show on a balance scale:



In order to solve this equation for J, we must find what balances J. To do this, we remove three "blocks" from each side of the balance scale:



This is what we have left:



We can express this algebraically as follows:

$J + 3 = 12$

$J + 3 - 3 = 12 - 3$

$J = 9$

Because we have now solved for J, we can go back and check the solution. Substituting $J = 9$ into the original equation $J + 3 = 12$ gives us $9 + 3 = 12$, which is correct. Jeremy's age is 9.

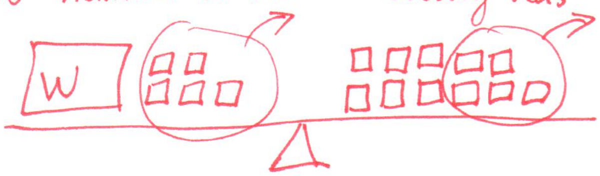
Write an equation to solve the next problem and model it on a balance scale.

EXAMPLE 5

If Wesley finds 5 more marbles, he will have the same number of marbles as John. John has 11 marbles. How many marbles does Wesley have?

W = number of marbles Wesley has

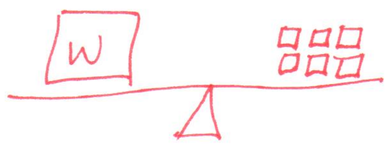
$W + 5 = 11$



$W + 5 - 5 = 11 - 5$

$W + 0 = 6$

$W = 6$



Wesley has 6 marbles

$W = 6$

EXPLORATION 3: SUBTRACTION PROPERTY OF EQUALITY

The rule we are using to solve these problems is called the **subtraction property of equality** because in each, we are subtracting the same number (removing the same number of blocks) from both sides of an equation. The new equation we obtain is said to be **equivalent** to the original equation because the two equations have the same solution: any value for a variable that makes one of the equations balance will make the other balance as well.

Definition 6.2: Subtraction property of Equality
If $A = B$ then $A - C = B - C$.

Consider the sequence 3, 4, 5, 6, … . We see that the term in a particular position, n, is $n + 2$. We can write this as the term = position + 2 or $t = n + 2$, where we let the variables t = term and n = position. According to this pattern or "rule", the 20th term would be 22 and the 200th term would be 202.

Suppose you were given the term 187. Can you determine which position this number would be in the list above?

term = position + 2

Remember that $t =$ $n + 2$, and 187 is the term. Symbolically, we could write $t =$ $n + 2$.

If we rewrite the equation substituting 187 for t, how can we determine n?

Write $187 = n + 2$ How could we use the subtraction property to solve?

subtract 2 from both sides.

Finding the value that makes an equation true is referred to as **solving an equation**. Solving equations is a very important part of doing algebra and subtraction property is an important tool for solving equations.

EXPLORATION 4: EQUATIONS AND SEQUENCES

The total cost of a meal for three people is \$51. If the three people agree to split the cost equally, what would each person's cost be? Write two equations that could be used to model the problem. You do not have to solve the problem. Use x to represent the cost each person pays. $3x = 51$ or $x = \frac{51}{3}$

You may have found that one equation is $3 \cdot x = 51$. Another equation could be written as $x = \frac{51}{3}$. You may recall this important connection between multiplication and division. We will talk more about this when we multiply by fractions in the next chapter.

We can relate the idea above with the sequence 3, 6, 9, 12, 15... Suppose we ask the question, "What position is 51 in this list?" We write this question mathematically as $3 \cdot x = 51$, where we let x represent the position the term 51 occupies in the list. From above, $3 \cdot x = 51$ is equivalent to $x = \frac{51}{3} = 17$. We conclude that 51 occupies the 17th position in the list above.

Try using the same strategy to solve the word problem below.

You and four friends have dinner at a nice restaurant. The bill comes to \$135, which you agreed to split evenly.

Write two equations that could be used to model the problem:

$5x = 135$ or $x = \frac{135}{5}$

If we think of the sequence 5, 10, 15, 20, … , we want to know what position 135 is in the list. Use an equation from above to solve. 135 occupies the 27th position in the sequence.

$x = \frac{135}{5}$ $135 \div 5 = 27$

$x = 27$

PROBLEMS

1. Translate the sentences below where x is a number.

   x less than 10 equals 8.

   a. $10 - x = 8$

   10 less than x equals 8.

   b. $x - 10 = 8$

2. Fill in the blanks to complete the steps to solve an equation.

   a. Step 1: Define your variable

   b. Step 2: Translate the problem into an equation

   c. Step 3: Solve for the unknown variable

   d. Step 4: Check your answer

3. Colton purchased half of a dozen candy bars from the school fundraiser. Robin purchased 5 times that amount. Write an equation to find how many candy bars Robin purchased. Remember to follow the steps listed above.

   R = Robin's candy bars $\quad 5(6) = R \quad R = 30 \quad 5 \cdot 6 = 30$ ✓

4. Write an equation for each scenario below and solve to find the unknown value.

   a. Martin has \$30. How much more does he need if he wants a total of \$113?

   $30 + x = 113 \quad 30 + x - 30 = 113 - 30 \quad x = 83$

   b. If BJ will be 19 in 7 years, how old is BJ now?

   $y + 7 = 19 \quad y + 7 - 7 = 19 - 7 \quad y = 12$

   c. Ben is 16 years old. In how many years will he be 34?

   $16 + y = 34 \quad 16 + y - 16 = 34 - 16 \quad y = 18$

   d. Brooke is 59 inches tall. How tall will she be after a 4-inch growth spurt?

   $59 + 4 = B \quad 63 = B$

   5 ft 3 in

5. Alison has a certain number of gel pens, let g represent the number of pens that Alison has. Pablo has 7 more pens than Alison, and Raquel has three times as many pens as Pablo. How many pens does Raquel have if Alison has: $P = g + 7$ $R = 3P \rightarrow R = 3(g+7) = 3g + 21$

a. 5 pens? 36

$P = (5) + 7 = 12$
$R = 3(12) = 36$

b. 10 pens? 51

$P = (10) + 7 = 17$
$R = 3(17) = 51$

c. g pens? $3g + 21$

$P = (g) + 7$
$R = 3(g + 7)$
$R = 3g + 21$

6. Write a story problem for this equation: $x - 32 = 75$

James has some chocolate chips. He eats 32 and discovers that he has 75 left. How many chocolate chips did he start with?

7. Solve the following equations using the 4-step process:

a. $m - 16 = 20$

m = variable
$m - 16 = 20$
$m - 16 + 16 = 20 + 16$
$m = 36$
$36 - 16 = 20$
✓

b. $16.2 + s = 45.095$

s = variable
$16.2 + s = 45.095$
$16.2 + s - 16.2 = 45.095 - 16.2$
$s = 28.895$
$16.2 + 28.895 = 45.095$
✓

c. $f - 3.01 = 12.4$

f = unknown variable
$f - 3.01 = 12.4$
$f - 3.01 + 3.01 = 12.4 + 3.01$
$f = 15.41$
$15.41 - 3.01 = 12.4$
✓

d. $14.7 + t = 14.9$

t = unknown
$14.7 + t - 14.7 = 14.9 - 14.7$
$t = 0.2$
$14.7 + 0.2 = 14.9$
✓

SUMMARY (What I learned in this section)