PATTERNS AND FUNCTIONS 5

Name: *Key*　　　　　Date:_____　　Period:_____

SECTION 5.3 FUNCTIONS

VOCABULARY

DEFINITION	EXAMPLE
Function: *A rule assigning a unique output to each input value*	$f(input) = output$ $f(x) = 2x$
Domain: *set of inputs*	$\{1, 3, 5, 6\}$ *with above function*
Range: *set of outputs*	$\{2, 6, 10, 12\}$
Function notation: *"the function pairs 1 with 2"* $f(x)$ *is not multiplication.* $f(x) = y$ *means function f sends x to y.*	$f(1) = 2$

Big Idea: What defines a function?

EXPLORATION 1

Sarah builds model airplanes. She makes two airplanes each day. How many airplanes will she make in 4 days? 10 days? Organize the information to reveal a pattern in the number of airplanes she makes in a given number of days.

days	0	1	2	3	4	5	6	7	8	9	10
airplanes	0	2	4	6	8	10	12	14	16	18	20

How did you organize your information in the exploration above? Describe the pattern that you see in the number of airplanes Sarah can make in a given number of days?

She can make 2 planes each day, so in x days, she can make 2x planes. The number of planes is double the number of days.

One way to organize such information is to build a table such as the one to the right. Notice that the first column is the number of days, and the second column is the total number of airplanes that Sarah can make in the corresponding number of days.

Days	Total Number of Planes
0	0
1	2
2	4
3	6
5	10
10	20
x	

What do you notice about this table? Why is this a good way to organize the information?

The table starts with 0 days. The left column increases by 1 at first and the right column increases by 2 at first, before the days skip.

What is the domain of the function in exploration 1? Explain.

*Non-negative numbers. Negative days do not make sense.*

What values are not in the domain? Explain.

Negatives. Negative days do not make sense.

What is the range of the function in exploration 1? Explain.

Non-negative numbers. A negative number of planes does not make sense. Any positive number of planes can be divided by 2 to find the number of days.

FUNCTION MACHINE

Input → **Rule** → **Output**
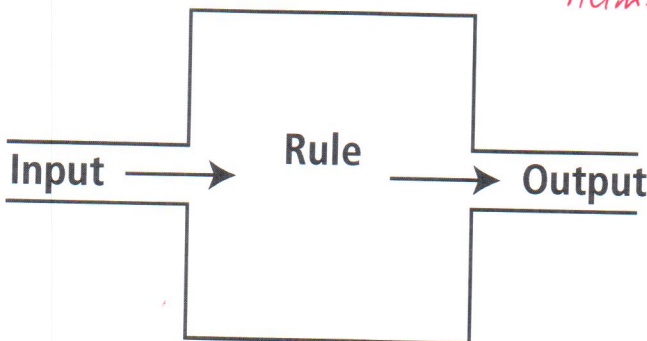
EXAMPLE 1

Let $F(x) = y = x + 4$

a. If the input is 2, what is the output F(2)? F(2) = ___6___

b. If the input is 4, what is the output F(4)? F(4) = ___8___

c. If the input is -1, what is the output F(-1)? F(-1) = ___3___

$$F(2) = y = (2) + 4$$
$$F(4) = y = (4) + 4$$
$$F(-1) = y = (-1) + 4$$

d. Make a two column table with inputs from $x = -4$ to $x = 4$.

x	
-4	0
-3	1
-2	2
-1	3
0	4
1	5
2	6
3	7
4	8

e. What input will yield an output of 20? In other words, for what x is $F(x) = 20$?

$$y = 20$$
$$20 = x + 4$$
$$20 - 4 = x + 4 - 4$$
$$\boxed{16 = x}$$

EXPLORATION 2

Juan sells peanuts for $4 per pound. Fill in the table below:

lbs of peanuts	Cost in dollars
0	$0
1	$4
2	$8
3	$12
4	$16
10	$40
26	$104
x	$C(x) = y = 4x$

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a. Write an equation for the cost C.

$$C(x) = 4x$$

b. Why is $x = -1$ not an input for this test function?

Buying -1 lbs of peanuts does not make sense.

c. Is $x = 1.5$ a possible input? Explain what is the value of $C(1.5)$?

Yes, you could buy a part of a lb of peanuts.
$C(1.5) = 1.5 \cdot 4 = 6$. So 1.5 lbs of peanuts costs $6

d. What is the domain of this cost function?

Any non-negative number.

e. What is the independent variable for this cost function? lbs of peanuts

f. What is the dependent variable for this cost function? cost in dollars of peanuts

PROBLEM 1

Mary pays $5 to get into the carnival. It costs $2 per ride. Make a table with the number of rides as inputs and the total cost as the output. What is the equation for the cost function?

number of rides	cost
0	5
1	7
2	9
3	11
4	13
10	25
x	5 + 2x

C = total cost

$$C(x) = y = 2x + 5$$

EXAMPLE 2

Fill in the function table for the function F given by the rule $F(x) = y = 2x - 1$

x	y	(x, y)
-4	$2(-4) - 1 = -9$	$(-4, -9)$
0	$2(0) - 1 = -1$	$(0, -1)$
2	$2(2) - 1 = 3$	$(2, 3)$
4	$2(4) - 1 = 7$	$(4, 7)$

Make a table for the function with inputs from -6 to +6. What can this new table tell us that the table above did not?

The steps from integer to integer are easier to see.

x	$F(x) = y$
-6	-13
-5	-11
-4	-9
-3	-7
-2	-5
-1	-3
0	-1
1	1
2	3
3	5
4	7
5	9
6	11

EXAMPLE 3

In Detroit, Michigan, the temperature rises an average of 3 °F per hour over a twelve-hour period from 7 a.m. to 7 p.m. The temperature at 3 p.m. is 36 °F. Let $T(x)$ represent the temperature in Detroit x hours after 3 p.m.

a. What does $x = 0$ and $x = -2$ mean in the context of this problem? What is $T(0)$, $T(1)$, and $T(-2)$?

   ↳ 3p.m.   → 1p.m. (2 hours before 3p.m.)

$$T(0) = 36$$
$$T(1) = 36 + 3 = 39$$
$$T(-2) = 36 - 3 - 3 = 30$$

b. What x-value corresponds to 11 a.m.? What is the temperature at 11 a.m.?

$$x = -4$$
$$36 - 3 - 3 - 3 - 3 = 24$$

c. Start to make a table of the time x from 7 a.m. to 7 p.m. and the corresponding temperature $T(x)$.

time	7am	8	9	10	11	12noon	1p.m	2	3	4	5	6	7
x	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
$T(x)$	12	15	18	21	24	27	30	33	36	39	42	45	48

d. Write the relationship between temperature and time using function notation. In other words, find an equation for $T(x)$.

$$T(x) = 36 + 3x$$

e. At what time during the twelve-hour period is the temperature equal to 27 °F? 42 °F?

$$27 = 36 + 3x$$
$$27 - 36 = 3x + 36 - 36$$
$$-9 = 3x$$
$$x = -3$$
the time is noon

$$42 = 36 + 3x$$
$$42 - 36 = 3x + 36 - 36$$
$$6 = 3x$$
$$x = 2$$
the time is 5p.m.

PRACTICE EXCERSIES

Complete the table for each function (rule). Show the work for the rule in the center column, plugging in for x.

1. $y = 5x$

x	$y = 5x$	y	(x,y)
1	$5(1) = 5$	5	$(1, 5)$
2	$5(2) = 10$	10	$(2, 10)$
3	$5(3) = 15$	15	$(3, 15)$
4	$5(4) = 20$	20	$(4, 20)$

3. $y = 3 - x$

x	$y = 3 - x$	y	(x,y)
0	$3 - 0 = 3$	3	$(0, 3)$
2	$3 - 2 = 1$	1	$(2, 1)$
4	$3 - 4 = -1$	-1	$(4, -1)$
6	$3 - 6 = -3$	-3	$(6, -3)$

2. $y = x + 8$

x	$y = x + 8$	y	(x,y)
0	$0 + 8 = 8$	8	$(0, 8)$
1	$+ 8 = 9$	9	$(1, 9)$
2	$2 + 8 = 10$	10	$(2, 10)$
3	$3 + 8 = 11$	11	$(3, 11)$

4. $y = 2x + 1$

x	$y = 2x + 1$	y	(x,y)
0	$2(0) + 1 = 1$	1	$(0, 1)$
1	$2(1) + 1 = 3$	3	$(1, 3)$
2	$2(2) + 1 = 5$	5	$(2, 5)$
3	$2(3) + 1 = 7$	7	$(3, 7)$

Determine the function rule for each data set and write y as an expression in terms of x.

5. $y = 2x - 3$

x	y	
0	-3	
1	-1	+2
2	1	+2
3	3	+2

7. $y = 10x$

	x	y	
	0	0	
+5	5	50	+50
+5	10	100	+50
+10	20	200	+100

6. $y = x + 2$

	x	y	
	-3	-1	
+1	-2	0	+1
+2	0	2	+2
+10	10	12	+10

8. $y = 2x + 5$

x	y	
0	5	
1	7	+2
2	9	+2
3	11	+2

SUMMARY (What I learned today)