PATTERNS AND FUNCTIONS 5

Name: Key Date:_____ Period:_____

SECTION 5.5 LINEAR FUNCTIONS

VOCABULARY

DEFINITION	EXAMPLE
Linear Function: a rule that produces a straight line graph: $y=mx+b$	$y=2x+2$
Constant rate of change: how quickly y decreases or increases when x changes by 1, slope	$m=\frac{1}{2}$, $y=\frac{1}{2}x+3$
***y*-intercept:** the point at which x is zero, b	$b=3$, $y=\frac{1}{2}x+3$
Constant of Proportionality: K when $y=Kx$ and $x=0$, so $K=\frac{y}{x}$.	$K=4$ $y=4x$

Big Idea: What is the general form of a linear equation?

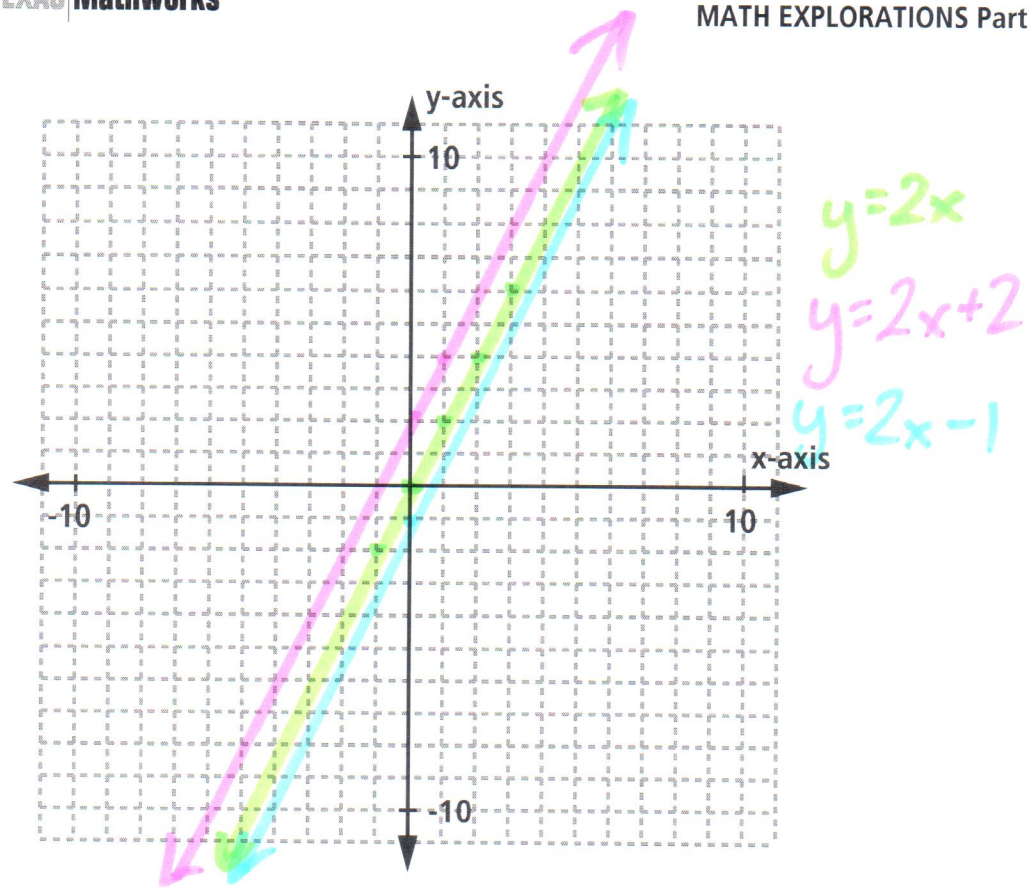
EXPLORATION 1

a. Fill in the table for each of these functions given in the table below. Plot these points and draw the lines.

b. What do the graphs of these lines have in common? slope, m value, they're parallel

c. What do their tables have in common? each y-value goes up by 2 for each x.

Input	Output		
x	$f(x)=y=2x$	$g(x)=2x+2$	$h(x)=2x-1$
-1	-2	0	-3
0	0	2	-1
1	2	4	1
2	4	6	3
3	6	8	5



The rules that produce straight line graphs all have a common form:

$$y = mx + b$$

The functions are called **linear functions**, that is, they have equations of the form $y = mx + b$.

Questions:

1. In Exploration 1, what is the value of m for each of the lines?

m=2 for all three lines

2. What effect does the number m have on the graph for each line?

graph: the slope is 2, they are parallel
table: input increases by 1 → output increases by 2.

3. What effect does the number m have on the graph for each line?

the slope is 2, they are parallel

4. What role does the number b and the *y*-intercept play for each line?

where x=0, or where the line intercepts the y-axis.

PROBLEM 1

Consider the following three linear functions:

a. $y = 3x - 2$ $b = -2$
b. $y = 3x + 4$ $b = 4$ $m = 3$
c. $y = 3x + 11$ $b = 11$

Predict how their graphs will be related. Use a graphing calculator to check your predictions.

They will be parallel. c will be on top, b will be in the middle, and a will be on the bottom.

EXPLORATION 2

Functions F and G are defined as $y_1 = F(x) = 4x$ and $y_2 = G(x) = x + 4$.

a. Fill in the table for F and G with $x = \{0, 1, 2, 3, 4, 5\}$.

x	$F(x) = y_1$	$y \div x$	$G(x) = y_2$	$y \div x$
-2	-8	4	2	-1
-1	-4	4	3	-3
0	0	X	4	X
1	4	4	5	5
2	8	4	6	3
3	12	4	7	$\frac{7}{3}$
10	40	4	14	$\frac{7}{5}$

b. Is there a pattern for the third column? For the fifth column? Explain.

Yes, $\frac{y}{x} = 4$ unless $x = 0$. No, no visible pattern.

If a function has a rule in the form $y = kx$, then for any input $x \neq 0$, the quotient of $\frac{y}{x}$ will always have the value k. The number k is called the **constant of proportionality**, and we say y is proportional to x.

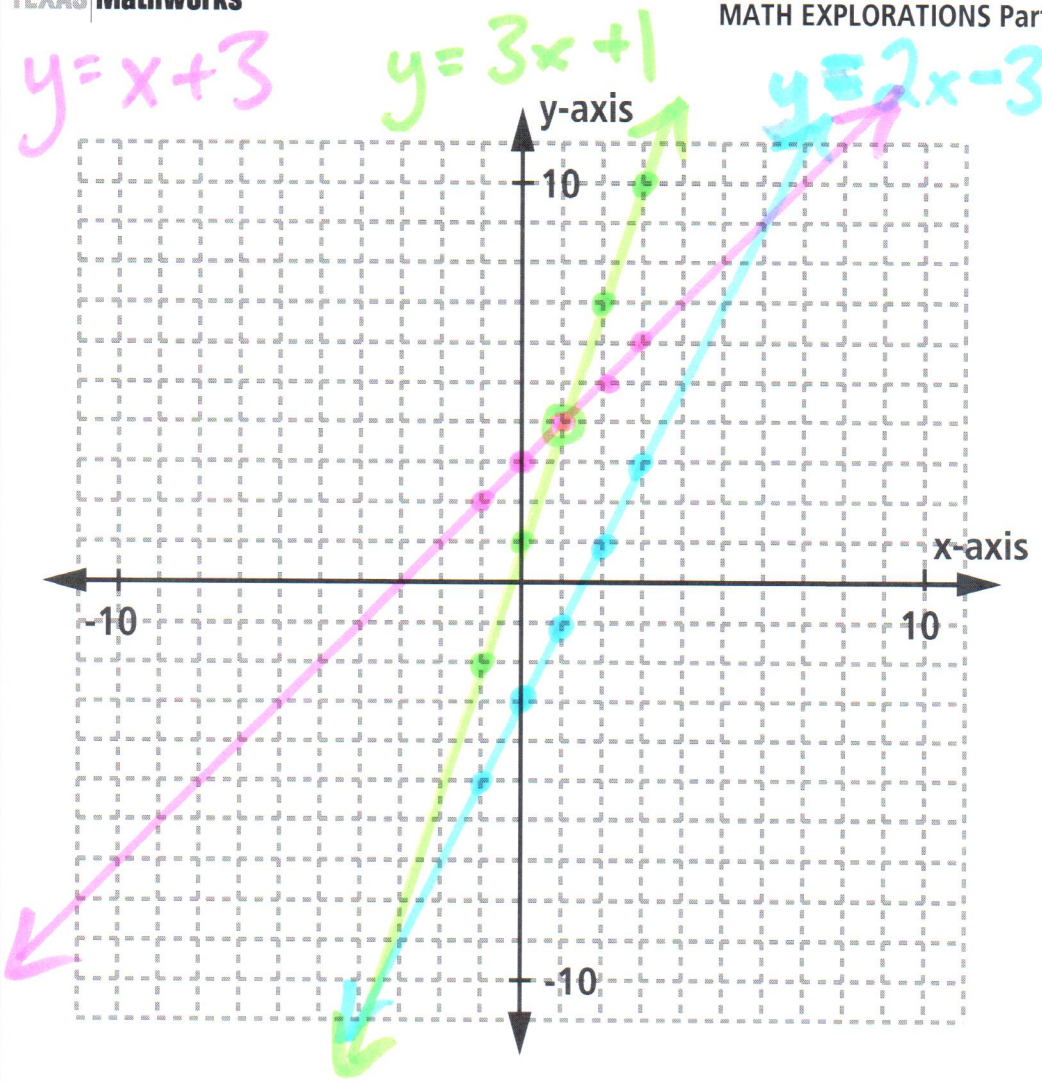
PRACTICE EXCERCISES

1. a. Fill in the table for each of the functions given below. Use the points to graph the functions.

Input	Output
x	$f(x) = x + 3$
-1	2
0	3
1	4
2	5
3	6

Input	Output
x	$g(x) = 3x + 1$
-1	-2
0	1
1	4
2	7
3	10

Input	Output
x	$h(x) = 2x - 3$
-1	-5
0	-3
1	-1
2	1
3	3



b. Identify the constant rates of change, m, and the y-intercepts, b, for each function.

$y = x + 3$ $m = 1$ $b = 3$

$y = 3x + 1$ $m = 3$ $b = 1$

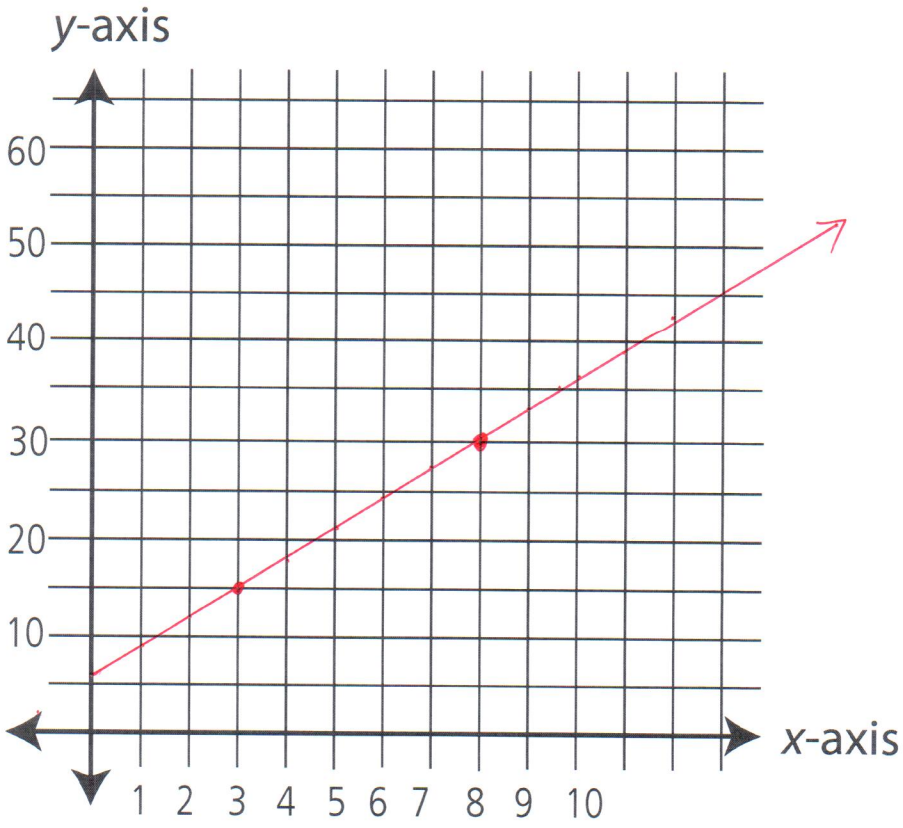
$y = 2x - 3$ $m = 2$ $b = -3$

2. Willow is making bracelets to sell for a school fundraiser. She spends $3 for supplies for each bracelet and plans to sell them for $5 each. She also had to spend $6 to advertise the fundraiser.

   a. Let x represent the number of bracelets Willow makes. Create a table listing total cost in dollars for the number of bracelets, $x = 0, 1, 2$ to 10.

x	$y = 3x + 6$
0	6
1	9
2	12
3	15
4	18
5	21
6	24
7	27
8	30
9	33
10	36

   b. Plot these points on a coordinate graph and draw a line to represent the total cost in relation to the number of bracelets made.



c. Write an equation for y, the total cost, in terms of the number of bracelets, x that Willow makes and sells.

makes: $y = 6 + 3x$ or $y = 3x + 6$

sells: $y = 5x$

d. What is the constant rate of change, m? What is the y-intercept, b?

makes: $m = 3$ $b = 6$

sells: $m = 5$ $b = 0$

e. Create a table listing Willow's revenue for $x = 0$ to $x = 10$ bracelets, and plot these points on the same graph as total cost.

revenue is (cost to sell) or earned amount

x	Revenue
0	0
1	5
2	10
3	15
4	20
5	25
6	30
7	35
8	40
9	45
10	50

f. At what point do the two lines intersect? What does this point represent?

$y = 3x + 6$ and $y = 5x$ intersect where

$3x + 6 = 5x$

$3x + 6 - 3x = 5x - 3x$

$6 = 2x$

$x = 3$

This is the breakeven point, (3, 15), where the cost to make and sell is the same. When Willow sells 3 bracelets, she will have spent and earned the same amount.

SUMMARY (What I learned today)