Name: Key Date:________ Period:______

SECTION 5.6 PATTERNS AND SEQUENCES

VOCABULARY

DEFINITION	EXAMPLE
Sequence: list of numbers, can be thought of as a functions with inputs starting at 1 or 0.	4, 8, 12, 16, ...
Term: member of a sequence	4 is the first term 8 is the second term
Arithmetic sequence: $a_1, a_2, a_3, \ldots$ is an arithmetic sequence if there is a c so that $a_{n+1} = a_n + c$	for 4, 8, 12, 16, ... $c = 4$.

or $a_{n+1} - a_n = c$

Big Idea: How do we identify the function rule that defines a sequence?

EXPLORATION 1

For the following list of numbers, determine what you think is the next number in the list. Explain how you made your choice.

a. 4, 8, 12, 16, <u>20</u>, ... add 4 each time

b. 1, 3, 5, 7, <u>9</u>, ... add 2 each time

c. 3, 6, 12, 24, <u>48</u>, ... multiply by 2 each time

EXAMPLE 1

Consider the sequence defined by the rule that for each input n, representing the place in the list, the output is given by $y = 2n$. Making a horizontal table of outputs, we get

Input	1	2	3	4	5	6	10	25	n
Output	2	4	6	8	10	12	20	50	$2n$

Thus the sequence can also be written as the list: 2, 4, 6, 8, ... Each member of a sequence is called a **term of the sequence**. The three dots at the end of the list means that the sequence continues for all positive integers. It is common to think of the first term in the list as the output you get with input of $n = 1$. If we use function notation, then the first term is denoted by a(1) or a_1. Organizing the sequence vertically, we get

$$a(1) = a_1 = 2$$
$$a(2) = a_2 = 4$$
$$a(3) = a_3 = 6$$
$$a(4) = a_4 = 8$$
$$a(n) = a_n = 2n$$
$$\ldots$$

PROBLEM 1

Based on the portion of the sequences shown below, write the next two terms in each sequence. Explain how you determined the pattern.

a. 3, 6, 9, 12, 15, 18, ... add 3 $(c = 3)$

b. 4, 7, 10, 13, 16, 19, ... add 3, $c = 3$

c. 2, 6, 10, 14, 18, 22, 26, ... add 4, $c = 4$

d. 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, ... add 1 or 2.
2 1 2 1 2 1 2
<u>Not</u> an arithmetic sequence.

PROBLEM 2

Determine a rule for each of the sequences in parts a through c in Problem 1.

a.
$a_1 = 3$

$a_2 = 3+3 = (2)(3)$

$a_3 = 3+3+3 = (3)(3)$

$\boxed{a_n = 3n}$

b. $a_1 = 4$

$a_2 = 4+3 = 7$

$a_3 = 4+3+3 = 10$

either
$4+(n-1)(3)$
$4+3n-3$

or
$\underbrace{1+3}_{4}+3+3 = 3n+1$

$\boxed{a_n = 3n+1 \quad \text{(or same)} \quad a_n = 3n+1}$

c. $a_1 = 2$

$a_2 = 4+2$

$a_3 = 4+4+2$

$a_n = 4(n-1)+2$

$a_n = 4n-4+2$

$\boxed{a_n = 4n-2}$

PROBLEM 3

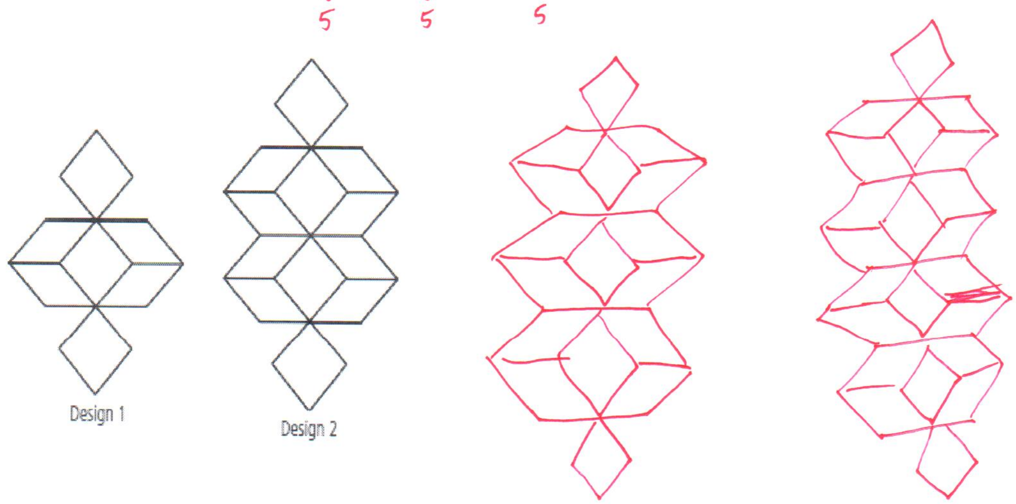
Use the pattern blocks to make the next two designs. Write a function rule that describes the sequence in terms of the design number *n*.

$a_n = 5n + 2$

each design has $5n$ s
and 2 s.

Design Numbers	1	2	3	4	5	6	7
Total Blocks	7	12	17	22	27	32	37

5 5 5



Design 1 Design 2

PRACTICE EXERCISES

1. Consider the sequence b defined by the rule $b(n) = 3n + 1$. Fill in the table with the first six terms.

Input	1	2	3	4	5	6
Output	4	7	10	13	16	19

2. Based on the following sequences shown below, write the next two terms. Write the corresponding rule for each sequence.

a. 8, 13, 18, 23, 28, 33, . . . a(*n*) = 5n + 3

b. 14, 20, 26, 32, 38, 44, . . . a(*n*) = 6n + 8

work shown:

a. $a_1 = 8 = 5 + 3$

$a_2 = 8 + 5 = (2)(5) + 3$

$a_n = 5n + 3$

b. $a_1 = 14 = 8 + 6$

$a_2 = 20 = 8 + 6 + 6$

$a_n = 8 + 6n$

SUMMARY (What I learned today)