LINEAR INEQUALITIES 5

SECTION 5.1 PROPERTIES OF INEQUALITIES

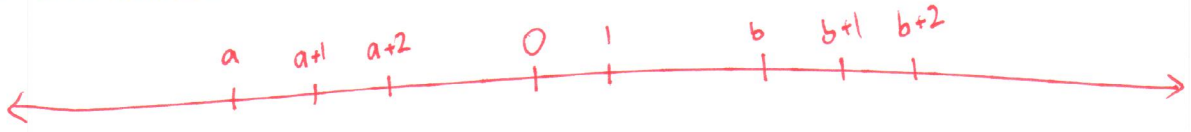
Name: Key **Date:** _____ **Period:** _____

Vocabulary

DEFINITION	EXAMPLE
Inequalities statements which compare 2 quantities	$3<6$ $x \geq -2$
Compound Inequalities Two inequalities, such as $x<2$ and $x>-5$, written in one statement	$-5<x<2$
Transitive Property of Inequality if $a<b$ and $b<c$ then $a<c$	$2<3$ and $3<500$ $2<500$
Subtraction Property of Inequality if $a<b$, $a-c<b-c$	$2<3$ $2-7<3-7$ $-5<-4$
Addition Property of Inequality if $a<b$, $a+c<b+c$	$-5<2$ $-5+3<2+3$ $-2<5$
Multiplication Property of Inequality if $a<b$: if $c>0$ $ac<bc$ if $c<0$ $ac>bc$	$-8<1$ $2(-8)<2(1)$ $-2(-8)>2(1)$ $-16<2$ $16>-2$
Division Property of Inequality if $a<b$: if $c>0$ $\frac{a}{c}<\frac{b}{c}$ if $c<0$ $\frac{a}{c}>\frac{b}{c}$	$-2<4$ $\frac{-2}{2}<\frac{4}{2}$ $\frac{-2}{-2}>\frac{4}{-2}$ $-1<2$ $1>-2$

EXPLORATION 1

1. Draw a number line. Be sure to locate and label 0 and 1.



2. Label two points on the number line a and b with a to the left of b. These can be any points you like as long as $a < b$.

3. Locate $a + 1$ and $b + 1$ on the number line. Is $a + 1 < b + 1$? How can you tell? Do you think it depends on your choice of a and b?

yes a+1 is left of b+1

no.

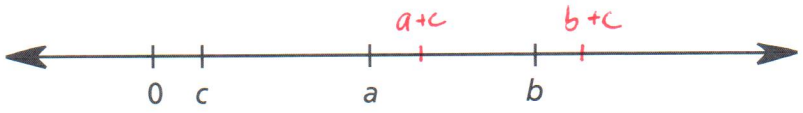
4. Locate $a + 2$ and $b + 2$ on the number line. Is $a + 2 < b + 2$? yes

EXPLORATION 2

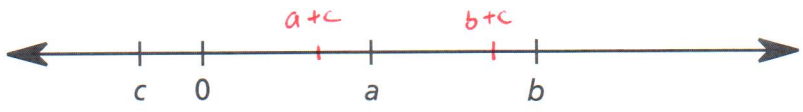
In this exploration, we will answer the following questions: Suppose we know that $a < b$. If c is a real number, then can we say that $a + c < b + c$? Is this statement always true? Is it sometimes true and sometimes false? Does it depend on the value of c?

1. Each of the following number lines depicts numbers a and b such that $a < b$. A number c is also shown. For each number line, locate $a + c$ and $b + c$ on the number line, and determine whether $a + c < b + c$.

a.



b.



2. If we know that $a < b$, is it always true that $a + c < b + c$? Does this depend on whether c is positive or negative?

yes

no

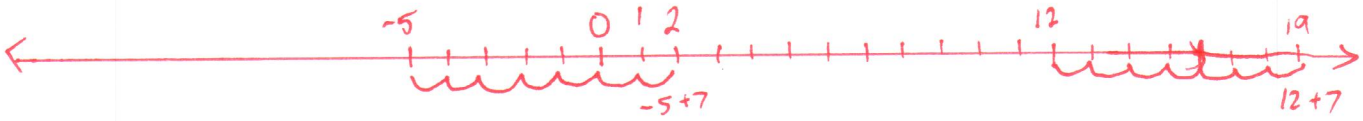
3. What do you notice about the distance between $a + c$ and $b + c$?

it is the same as the distance from a to b.

PROBLEM 1

Draw a number line. Locate and label 0, 1, −5, and 12. Use the number line to illustrate why if $-5 < 12$ then $-5 + 7 < 12 + 7$. Explain.

2<19



-5+7 moves right by 7, but so does 12+7 (they move by the same amount)

PROBLEM 2

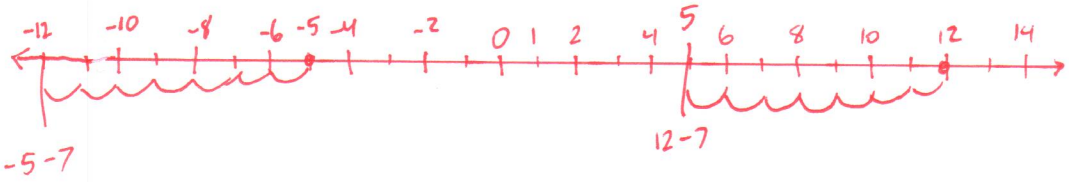
Suppose that a, b, and c are numbers such that $a > b$. Is it true that $a + c > b + c$? Explain.

yes. This is the same as b<a. So b+c<a+c, which is the same as a+c>b+c.

The distance from a to b is the same as the distance from a+c to b+c

PROBLEM 3

Draw a number line. Locate and label 0, 1, −5, and 12. Use the number line to illustrate why if $-5 < 12$ then $-5 - 7 < 12 - 7$. Explain.



-5<12

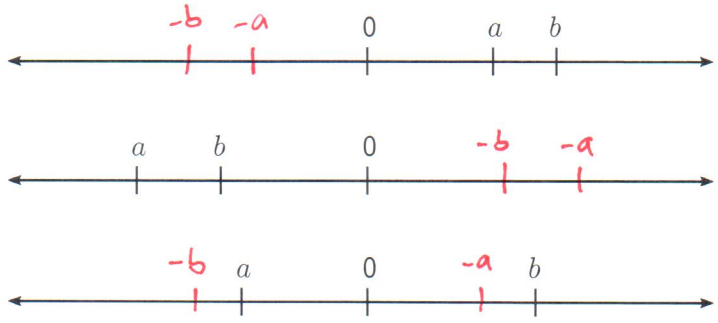
-5-7<12-7

-12<5

EXPLORATION 3

Suppose a and b are numbers so that $a < b$.

1. Using the three possible locations for a and b below, plot the locations of the points representing $-a$ and $-b$ in each case:

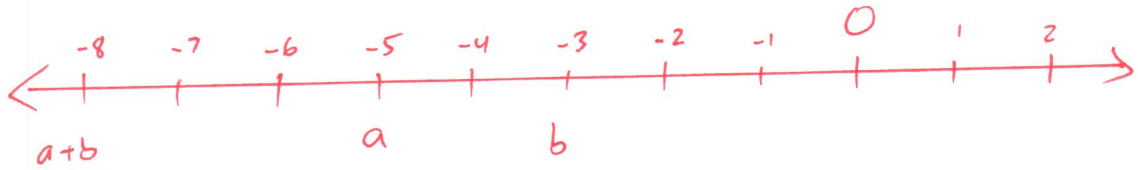


2. If $a < b$, make a rule for the relationship between $-a$ and $-b$. Justify your rule.

$-a > b$ or $-b < a$. $-a$ moves to the right of $-b$ if a is left of b ($a < b$).

PROBLEM 4

Find an example of a pair of numbers a and b for which each of the following is true: $a < b$ and $a + b < a$. Illustrate that your answer is correct on the number line using exact numerical values. Don't forget to mark 0 on the number line.



if $a = -5$ and $b = -3$

$a < b$ $-5 < -3$

$a + b < a$ $-5 + -3 < -5$

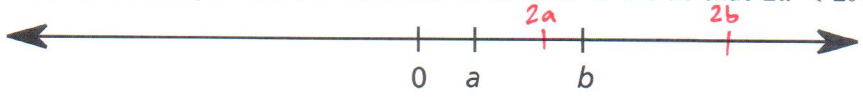
$-8 < -5$

We have addition and subtraction properties of inequality. Are there also multiplication and division properties of inequality? Exploration 3 tells we must be careful when dealing with negatives.

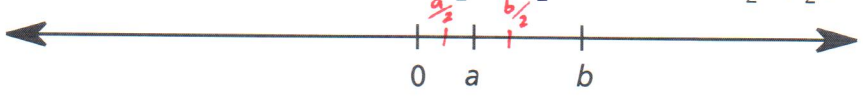
EXPLORATION 4

The following number line depicts a situation in which $a < b$:

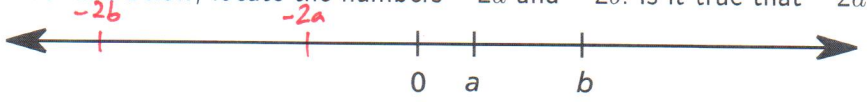
1. On the number line below, locate the numbers $2a$ and $2b$. Is it true that $2a < 2b$? yes



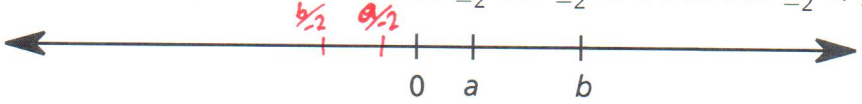
2. On the number line below, locate the numbers $\frac{a}{2}$ and $\frac{b}{2}$. Is it true that $\frac{a}{2} < \frac{b}{2}$? yes



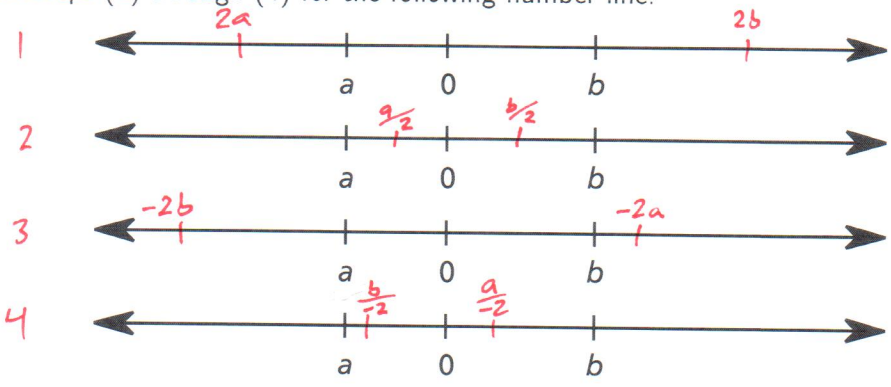
3. On the number line below, locate the numbers $-2a$ and $-2b$. Is it true that $-2a < -2b$? no



4. On the number line below, locate the numbers $\frac{a}{-2}$ and $\frac{b}{-2}$. Is it true that $\frac{a}{-2} < \frac{b}{-2}$? no.



5. Repeat steps (1) through (4) for the following number line:



2 times distance between a & b

6. What do you notice about the distance between $2a$ and $2b$? How about the distance between $\frac{a}{2}$ and $\frac{b}{2}$? The distance between $-2a$ and $-2b$? The distance between $\frac{a}{-2}$ and $\frac{b}{-2}$?

$\frac{1}{2}$ distance between a and b

2 times distance between a and b

$\frac{1}{2}$ distance between a and b.

EXAMPLE 1

Suppose that a and b are numbers such that $a < b$. For each of the following statements, determine whether the statement is *always true*, *always false*, or *could be true or false, depending on the values of* a *and* b.

1. $a < b + 1$ always true
2. $a + 1 < b$ depends on values of a and b.
3. $a + 5 < b + 5$ always true
4. $3a < 3b$ always true
5. $-a < -b$ always false
6. $a + b < 2b$ always true (since $a+b < b+b$)
7. $7a - 10 < 7b - 10$ always true: $7a < 7b$ so $7a - 10 < 7b - 10$

PROBLEM 5

Suppose that a and b are numbers such that $a < b$. For each of the following statements, determine whether the statement is *always true*, *always false*, or *could be true or false, depending on the values of* a *and* b.

1. $-3a > -3b$ always true
2. $2a < 4b$ depends on values of a and b
3. $10 - a < 10 - b$ always false: $a < b$ $-a > -b$ so $10 - a > 10 - b$
4. $ab < b^2$ depends on if b is positive.

SUMMARY (What I learned today)