

SECTION 5.2 SOLVING LINEAR INEQUALITIES

Name: Key Date: _____ Period: _____

EXPLORATION 1

In her algebra class, Olivia’s homework grade is 94, and her quiz grade is 91. She has a test coming up; her average in the class will be the average of her grade on the test, her homework grade, and her quiz grade. Olivia wants to earn an average of at least 90 so that she will have an A in the class.

- Let T represent Olivia’s grade on the upcoming test. Write an expression, in terms of T , for Olivia’s average in her algebra class.

$$\frac{T + 94 + 91}{3}$$

- Write the statement “Olivia’s average in her algebra class is at least 90” as an inequality in terms of T .

$$\frac{T + 94 + 91}{3} \geq 90$$

- Use the properties of inequalities from 5.1 to rewrite this inequality so that T alone is on the left side of the inequality.

$$\left(\frac{T + 94 + 91}{3}\right) 3 \geq 90 \cdot 3$$

- What grade does Olivia need to earn in order to make an A in the class?

$$T + 185 \geq 270$$

$$T \geq 85$$

Olivia needs to make at least an 85 to make an A in the class.

EXAMPLE 1

Solve the inequality $2x < 8$.

$$\frac{2x}{2} < \frac{8}{2} \quad \text{since } 2 > 0.$$

$$x < 4$$



EXPLORATION 2

One day in Weatherford, Texas, the temperature at 12:00 P.M. was 45 degrees Fahrenheit. A cold front was coming in, and the temperature dropped at a steady rate of 2 degrees per hour until midnight. In this Exploration, we'll determine during what part of the day the temperature was below freezing (32 degrees Fahrenheit).

- Let t be the number of hours that have passed since noon. Write an expression, in terms of t , for the temperature at time t . $45 - 2t$

- What inequality do we need to solve in order to determine when the temperature was below freezing? $45 - 2t < 32$

- Use the properties of inequality to solve this inequality. Be sure to state what the solution set of the inequality is. $45 - 2t - 45 < 32 - 45$
 $-2t < -13$
 $\frac{-2t}{-2} > \frac{-13}{-2}$
 $t > 6.5$ $\{t | t > 6.5\}$

- During what part of the day was the temperature in Weatherford below freezing?
After 6.5 hours after noon. That is, after 6:30 p.m.

PROBLEM 1

Solve the following inequalities. In each case, give the solution set of the inequality.

- $2x < 5$ $\frac{2x}{2} < \frac{5}{2}$ $x < \frac{5}{2}$ or $x < 2.5$ $\{x | x < 2.5\}$

- $2x + 3 < 13$ $2x + 3 - 3 < 13 - 3$ $2x < 10$ $\frac{2x}{2} < \frac{10}{2}$ $\{x | x < 5\}$

- $1 - 3x < 10$ $1 - 3x - 1 < 10 - 1$ $-3x < 9$ $\frac{-3x}{-3} > \frac{9}{-3}$ $\{x | x > -3\}$

- $4x + 8 < 2x + 2$ $4x + 8 - 8 < 2x + 2 - 8$ $4x < 2x - 6$ $4x - 2x < 2x - 6 - 2x$
 $2x < -6$ $\frac{2x}{2} < \frac{-6}{2}$ $x < -3$ $\{x | x < -3\}$

EXAMPLE 2

A bakery makes apple pies and sells them for \$8 each. Sales from the pies range between \$544 and \$664. How many pies does it take to produce this range of sales?

$x = \text{number of pies}$

$$544 \leq 8x$$

$$\frac{544}{8} \leq \frac{8x}{8}$$

$$68 \leq x$$

$$8x \leq 664$$

$$\frac{8x}{8} \leq \frac{664}{8}$$

$$x \leq 83$$

between 68 and 83,
inclusive

$$68 \leq x \leq 83$$

PROBLEM 2

Jacob has between \$3.85 and \$4.55 in nickels in his piggy bank. How many nickels are there in his bank?

$n = \text{number of nickels}$

$$3.85 \leq 0.05n$$

$$\frac{3.85}{0.05} \leq \frac{0.05n}{0.05}$$

$$77 \leq n$$

$$0.05n \leq 4.55$$

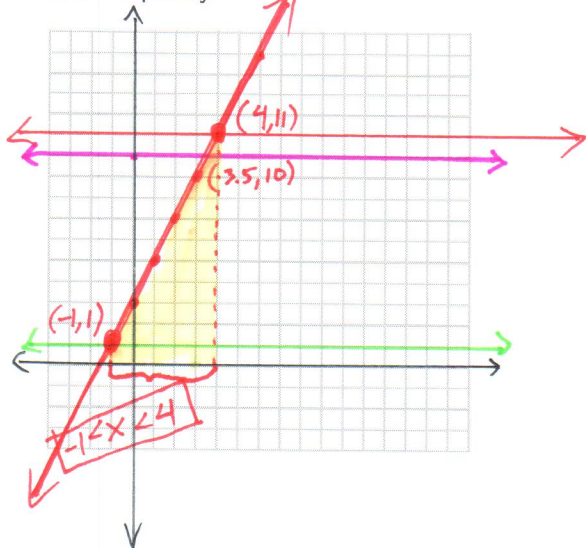
$$\frac{0.05n}{0.05} \leq \frac{4.55}{0.05}$$

$$n \leq 91$$

$$77 \leq n \leq 91$$

EXPLORATION 3

Graph the three lines given by equations below. Find the points of intersection. Then solve the inequality: $1 < 2x + 3 < 11$. Discuss the connection between the graphs and the solution set for the inequality.



- $y = 11$
- $y = 10$
- $y = 2x + 3$
- $y = 1$

$$1 < 2x + 3 < 11$$

$$1 < 2x + 3 \qquad 2x + 3 < 11$$

$$1 - 3 < 2x + 3 - 3 \qquad 2x + 3 - 3 < 11 - 3$$

$$-2 < 2x \qquad 2x < 8$$

$$-\frac{2}{2} < \frac{2x}{2} \qquad \frac{2x}{2} < \frac{8}{2}$$

$$-1 < x \qquad x < 4$$

$$-1 < x < 4$$

EXAMPLE 3

Charlie is hauling 70 identical pieces of steel pipe from Dallas to Austin on a trailer truck that weighs exactly 18500 pounds when it is empty. He has his truck weighed on a truck scale; the scale says that the truck weighs 29075 pounds. Charlie knows that his weight is 194 pounds, and the scale is accurate to within a margin of ten pounds. What is the range of possible values for the weight of one of the pieces of pipe?

w is the weight of a pipe

margin of 10 pounds:
weight is between
29065 and 29085
(29075-10) (29075+10)

$$70w + 18500 + 194$$

$$29065 < 70w + 18694$$

$$70w + 18694 < 29085$$

$$29065 - 18694 < 70w + 18694 - 18694$$

$$70w + 18694 - 18694 < 29085 - 18694$$

$$10371 < 70w$$

$$70w < 10391$$

$$\frac{10371}{70} < \frac{70w}{70}$$

$$\frac{70w}{70} < \frac{10391}{70}$$

$$148.1 < w$$

↑
underestimated

$$w < 148.5$$

↑
overestimated

$$\text{So } 148.1 < w < 148.5.$$

The pipe weighs between 148.1 and 148.5 pounds.

EXPLORATION 4

Suppose we want to find the solution set of the inequality

$$4 - 3x < 2x + 5 \leq 12 - x.$$

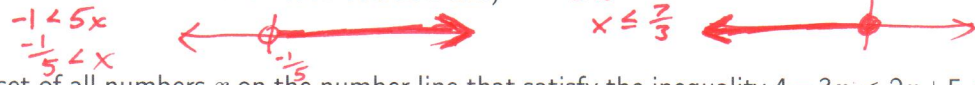
1. Try using properties of inequalities to isolate x on one "side" of the inequality, so that one "side" is simply x and the other two "sides" are numbers.

Not possible.

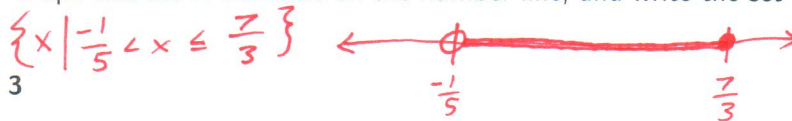
2. Write this inequality as two separate inequalities.

$$4 - 3x < 2x + 5 \qquad 2x + 5 \leq 12 - x$$

3. Find the solution set for each of these inequalities. Graph each solution set on a number line. (Use a different number line for each solution set.)



4. Find the set of all numbers x on the number line that satisfy the inequality $4 - 3x < 2x + 5 \leq 12 - x$. Graph this set of numbers on the number line, and write the set using set notation.



PROBLEM 3

Solve the following inequalities. Graph the solution set on the number line.

$$1. \quad 4 < 2x - 2 < 9$$

$$\frac{6}{2} < \frac{2x}{2} < \frac{11}{2} \quad \{x \mid 3 < x < \frac{11}{2}\}$$

$$2. \quad x \leq 3x - 5 < x + 20$$

$$0 \leq 2x - 5 < 20 \qquad \frac{5}{2} \leq \frac{2x}{2} < \frac{25}{2} \quad \left\{ \frac{5}{2} \leq x < \frac{25}{2} \right\}$$

$$3. \quad 8 - x \leq 3x \leq 5x - 2$$

$$3. \quad 8 - x \leq 3x \qquad 3x \leq 5x - 2$$

$$4. \quad 5x - 9 < 2x < 4x - 12$$

$$\frac{8 \leq 4x}{4} \qquad 2 \leq x \qquad 0 \leq 2x - 2 \qquad \frac{2 \leq 2x}{2} \qquad 1 \leq x$$

$$4. \quad 5x - 9 < 2x \qquad 2x < 4x - 12$$

EXAMPLE 4

Find all numbers x such that $5(x - 1) < -5$ or $3(x + 2) > 12$.

$$5(x - 1) < -5$$

$$5x - 5 < -5$$

$$\frac{5x}{5} < \frac{0}{5}$$

$$x < 0$$

$$\frac{3(x + 2)}{3} > \frac{12}{3}$$

$$x + 2 > 4$$

$$x > 2$$

$$3 > x \qquad x > 6$$

$$x < 3 \text{ and } x > 6$$

$$\boxed{\text{no solution}} \quad \emptyset$$



SUMMARY (What I learned today)
