NUMBER THEORY 7

Name: Key Date: Period:

SECTION 7.1 DIVISIBILITY, FACTORS AND MULTIPLES

VOCABULARY

DEFINITION	EXAMPLE
Divisible: n is divisible by d if there is an integer q so that $n = d \cdot q$ (n, d are integers, $d \neq 0$)	15 is divisible by 3 $15 = 3 \cdot 5$
Multiples: n is a multiple of d (see above)	8 is a multiple of 4 $8 = 4 \cdot 2$
Factors: d is a factor of n (see above	6 is a factor of 24 $24 = 6 \cdot 4$

Big Idea: How are factors and multiples related?

EXPLORATION 1: The Possible Rectangle Model

1. Complete the table provided in this packet. The first column contains the positive integer *n*. In the second column, write the number of rectangles possible with area *n*. In the third column, list all the possible dimensions of the rectangles. In the fourth column, list all the possible lengths of sides of the rectangles, in increasing order. For example, we have filled in the results for the value of $n = 4$ on the table.

2. What do you notice in the table so far? see page 153-154

Answers will vary. Most numbers have an even number of side lengths. Side lengths are factors of n. Some numbers (primes) have only 1 rectangle

3. Continue the table for *n* from 21 to 40. see page 154

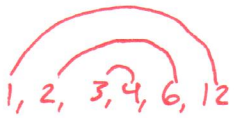
4. Looking at the extended table, do the patterns continue?

yes.

5. Looking at a given number *n*, what do you notice about the numbers in the last column for this value of *n*? The last column contains a list of all factors of n.

6. What do we call the numbers in the last column in relation to n? For each rectangle, the dimensions form a factor pair, such as 3 and 6 for $n = 18$. If you put all the factors in the last column in order, such as 1, 2, 3, 4, 6, 12 for $n = 12$, how do the factor pairs line up?

Factors/divisors. First and last line up, etc.



"Factor rainbow"

7. What do you notice about the number 1? Find any other numbers that have this same property, if possible.

Only 1 rectangle, only 1 factor, odd number of factors.

↓ primes — none — ↓ squares

8. Circle the values of n that generate only one rectangle. How many factors does each of these have?

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 (see 153-154)

How would you describe the circled numbers, excluding 1?

primes: have only 2 factors: itself and 1

9. Use a different color pen or marker to box the values of n that have an odd number of positive factors.

□ see 153-154

EXAMPLE 1

Is 13 a factor of the number 798? Equivalently, is 798 a multiple of 13?

no. no.

$793 = 13 \cdot 61$

$806 = 13 \cdot 62$

PROBLEM 1

a. Is the number 825 divisible by 15?

$825 = 15 \cdot 55$ yes

b. If a number is divisible by 15, is it also divisible by 3 and divisible by 5? Explain. yes.

If $n = 15 \cdot q$ (n is divisible by 15)

then $n = 3 \cdot 5 \cdot q$

$n = 3(5q)$ and $n = 5(3q)$ so n is divisible by 3 and 5.

n	Number of Possible Rectangles	Possible Rectangle Dimensions	Possible Side Lengths (in increasing order)
1	1	1×1	1
2	1	1×2	1, 2
3	1	1×3	1, 3
4	2	1 x 4, 2 x 2	1, 2, 4
5	1	1×5	1, 5
6	2	1×6, 2×3	1, 2, 3, 6
7	1	1×7	1, 7
8	2	1×8 2×4	1, 2, 4, 8
9	2	1×9 3×3	1, 3, 9
10	2	1×10 2×5	1, 2, 5, 10
11	1	1×11	1, 11
12	3	1×12 2×6 3×4	1, 2, 3, 4, 6, 12
13	1	1×13	1, 13
14	2	1×14 2×7	1, 2, 7, 14
15	2	1×15 3×5	1, 3, 5, 15
16	3	1×16 2×8 4×4	1, 2, 4, 8, 16
17	1	1×17	1, 17
18	3	1×18 2×9 3×6	1, 2, 3, 6, 9, 18
19	1	1×19	1, 19
20	3	1×20 2×10 4×5	1, 2, 4, 5, 10, 20

n	Number of Possible Rectangles	Possible Rectangle Dimensions	Possible Side Lengths (in increasing order)
21	2	1×21 3×7	1, 3, 7, 21
22	2	1×22 2×11	1, 2, 11, 22
23	1	1×23	1, 23
24	4	1×24 2×12 3×8 4×6	1, 2, 3, 4, 6, 8, 12, 24
25	2	1×25 5×5	1, 5, 25
26	2	1×26 2×13	1, 2, 13, 26
27	2	1×27 3×9	1, 3, 9, 27
28	3	1×28 2×14 4×7	1, 2, 4, 7, 14, 28
29	1	1×29	1, 29
30	4	1×30 2×15 3×10 5×6	1, 2, 3, 5, 6, 10, 15, 30
31	1	1×31	1, 31
32	3	1×32 2×16 4×8	1, 2, 4, 8, 16, 32
33	2	1×33 3×11	1, 3, 11, 33
34	2	1×34 2×17	1, 2, 17, 34
35	2	1×35 5×7	1, 5, 7, 35
36	5	1×36 2×18 3×12 4×9 6×6	1, 2, 3, 4, 6, 9, 12, 18, 36
37	1	1×37	1, 37
38	2	1×38 2×19	1, 2, 19, 38
39	2	1×39 3×13	1, 3, 13, 39
40	4	1×40 2×20 4×10 5×8	1, 2, 4, 5, 8, 10, 20, 40

EXPLORATION 2: SIEVE OF ERATOSTHENES

This exploration is based on an ancient method attributed to a famous Greek mathematician, Eratosthenes of Cyrene. The process involves letting certain kind of numbers pass through the sieve leaving other kinds of numbers in the sieve. Try the exploration and see for yourself.

1. Use the grid of the first 100 natural numbers in the rows of ten handout. ✓
2. Mark out the number 1. We will see why in the next section. ✓
3. Using a colored pencil or marker, circle the number 2 and then mark out every remaining multiple of 2 until you have gone through the whole list. What is a mathematical term for the marked out numbers? ✓
4. From the beginning, with a different colored pencil or marker, circle the first number that is not marked out and not circled. Then mark out all remaining multiples of that number. ✓
5. Repeat this process until you have gone all the way through the list. done for 2-5 on pg 156
6. Make a new ordered list of all the circled numbers. What do these numbers have in common? How is this list of numbers related to patterns from the possible rectangle activity? same as "1 rectangle" numbers
   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
7. You might have noticed that in the third round, some of the multiples of 3 were already crossed out in the second round. Find 3 such numbers. Why did this happen? They are multiples of 2 and 3.
   6, 18, 48
8. What kind of numbers did you mark out exactly twice? How many factors do these numbers have? An even number unless it is a square.
   Numbers with 2 prime factors.
9. What kind of numbers did you mark out once?
   Numbers that have only 1 prime factor. They are powers of a single prime.
10. After what number do we just circle the rest of the numbers on the list? Explain why we can do this.
    47. 48-50 are crossed out, and anything above 50 is not a factor of any number below 100.

Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

SUMMARY (What I learned today)