# NUMBER THEORY 7

**Name:** Key **Date:**________ **Period:**______

## SECTION 7.3 EXPONENTS AND ORDER OF OPERATIONS

### VOCABULARY

| DEFINITION | EXAMPLE |
|---|---|
| **Exponential notation:** Multiplication of a repeated factor written with an exponent | $2 \cdot 2 \cdot 2 = \boxed{2^3}$ |
| **Power or Exponent:** Number of times a factor is multiplied | $x^n$ |
| **Base:** A factor that is repeated | $x^n$ |
| **Order of operations:** the order that mathematical operations are performed | Parentheses<br>Exponential expressions<br>Mult./Div. left to right<br>Add./Subtr. left to right |

**Big Idea:** What are exponents and how are they used with respect to order of operations?

### EXAMPLE 1

Escherichia coli, more commonly known as E. coli, is a form of bacteria. If one of the bacteria lives in a petri dish and doubles each hour, how many bacteria will be in the dish after 1 hour? 2 hours? 3 hours? 5 hours? *n* hours?

$1 \cdot 2 = 2 = 2^1$ after 1 hour

$2 \cdot 2 = 4 = 2^2$ after 2 hours

$2 \cdot 2 \cdot 2 = 8 = 2^3$ after 3 hours

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$ after 5 hours

$2^n$ after n hours

## EXPLORATION 1

By using the definition of exponential notation and multiplication, we see that:

$$3^4 \cdot 3^6 = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) = 3^{10} = 3^{4+6}.$$

Compute the following products, showing all your work.

a. $3^2 \cdot 3^3$

$(3 \cdot 3)(3 \cdot 3 \cdot 3)$

$\boxed{3^5} = 3^{2+3}$

b. $3^3 \cdot 3^2$

$(3 \cdot 3 \cdot 3)(3 \cdot 3)$

$\boxed{3^5} = 3^{3+2}$

c. $2^4 \cdot 2^3$

$(2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$

$\boxed{2^7} = 2^{4+3}$

d. $10^3 \cdot 10^5$

$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$

$\boxed{10^8} = 10^{3+5}$

Does this calculation agree with your rule? Use this pattern to check your answers in Exploration 1. ✓

Yes.

Do you see a pattern for multiplying numbers in exponential form with the same base? Use your observation to determine the product in the problem below.

$$X^n \cdot X^m = X^{(n+m)}$$

## PROBLEM 1

Compute the product: $2^5 \cdot 2^4$

$$2^5 \cdot 2^4 = 2^{5+4} = \boxed{2^9} = \boxed{512}$$

What is the rule for multiplying numbers in exponential format with the same base?

$$X^a \cdot X^b = X^{(a+b)}$$

## PROBLEM 2

Rewrite the following products and then compute them. Use a calculator to compute both equivalent forms.

a. $4^3 \cdot 4^5 = 64 \cdot 1024 = 65536$
$4^{3+5} = 4^8 = 65536$

c. $10^3 \cdot 10^4 = 1{,}000 \cdot 10{,}000 = 10{,}000{,}000$
$10^{3+4} = 10^7 = 10{,}000{,}000$

b. $2^5 \cdot 2^5 = 32 \cdot 32 = 1024$
$2^{5+5} = 2^{10} = 1024$

d. $3 \cdot 3^3 = 3 \cdot 27 = 81$
$3^1 \cdot 3^3 = 3^{1+3} = 3^4 = 81$

## SPECIAL CASES: What do $4^1$ and $4^0$ equal?

We note that $4 \cdot 4 = 4^2 = 4^{1+1} = 4^1 \cdot 4^1$, therefore $4^1$ must be the same as 4. We can use the same process for any number x: $x \cdot x = x^2 = x^{1+1} = x^1 \cdot x^1$.

$x^1 = x$.

What does $4^0$ equal? Because $4 \cdot 4^0 = 4^1 \cdot 4^0 = 4^{1+0} = 4^1 = 4 = 4 \cdot 1$, we see that multiplying by $4^0$ is the same as multiplying by the number 1. We therefore assume that for any positive integer n.

$n^0 = 1$.

## PLACE VALUE

Consider the number 4,638. Using place value and our notation of exponents, we can rewrite 4,638 in the expanded notation:

$$4 \cdot 1{,}000 + 6 \cdot 100 + 3 \cdot 10 + 8 \cdot 1 = 4 \cdot 10^3 + 6 \cdot 10^2 + 3 \cdot 10^1 + 8 \cdot 10^0$$

Conversely we can start with the expression $4 \cdot 10^3 + 6 \cdot 10^2 + 3 \cdot 10^1 + 8 \cdot 10^0$, or in calculator notation,

$4 * 10^3 + 6 * 10^2 + 3 * 10^1 + 8 * 10^0$. In what order can we perform the calculations in this expression so the sum equals 4,638?

## PROBLEM 3

Write the following numbers in expanded notation:

a. $793 = 7 \cdot 100 + 9 \cdot 10 + 3 \cdot 1 = 7 \cdot 10^2 + 9 \cdot 10^1 + 3 \cdot 10^0$

b. $2843 = 2 \cdot 10^3 + 8 \cdot 10^2 + 4 \cdot 10^1 + 3 \cdot 10^0$

c. $3047 = 3 \cdot 10^3 + 0 \cdot 10^2 + 4 \cdot 10^1 + 7 \cdot 10^0$

**PROBLEM 4**

Compute the following:

a. $\underline{7} \cdot 10^2 + \underline{3} \cdot 10^1 + \underline{2} \cdot 10^0$ = 732

b. $\underline{6} \cdot 10^3 + \underline{5} \cdot 10^2 + \underline{9} \cdot 10^1 + \underline{3} \cdot 10^0$ = 6593

c. $\underline{6} \cdot 10^3 + \underline{9} \cdot 10^1 + \underline{3} \cdot 10^0$ = 6093

**EXPLORATION 2**

Compute the following, showing all your work. Review **Order of Operations** before you begin.

$20 - 10 \div 2 + 3^3 - 9$

$20 - 10 \div 2 + 27 - 9$

$20 - 5 + 27 - 9$

$\to 20 - 5 + 27 - 9$

$15 + 27 - 9$

$42 - 9$

$\to 42 - 9$

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*There are different forms of grouping symbols: Parentheses, ( ), brackets, [ ], and braces, { }. Absolute value symbols | | are also treated as a type of grouping symbol.*

**PROBLEM 5**

Compute the following:

a. $6 \div 2(4^2 + 7)$

$6 \div 2(16 + 7)$

$6 \div 2(23)$

$3 \cdot (23)$

$\boxed{69}$

b. $4 \cdot |7 - 3| \div 2$

$4 \cdot |4| \div 2$

$4 \cdot 4 \div 2$

$16 \div 2$

$\boxed{8}$

c. $4 + 2^3 \cdot 3 - (17 - 5) \cdot 3 + (17 - 5) \div 2$

$4 + 8 \cdot 3 - (12) \cdot 3 + (12) \div 2$

$4 + 24 - 36 + 6$

$28 - 36 + 6$

$-8 + 6 = \boxed{-2}$

d. $10 + (5 - 2^2) \cdot |-9 + 8|$

$10 + (5 - 4) \cdot |-1|$

$10 + (1) \cdot 1$

$10 + 1 = \boxed{11}$

**PRACTICE EXERCISES**

1. Evaluate the following expressions:

a. $2^3 \cdot 2^4 = 2^{3+4} = 2^7 = \boxed{128}$

b. $3^2 \cdot 3^4 = 3^{2+4} = 3^6 = 729$

c. $3^6 = \boxed{729}$

2. Write the following numbers in expanded notation

a. $386 = 3 \cdot 10^2 + 8 \cdot 10^1 + 6 \cdot 10^0$

b. $5{,}279 = 5 \cdot 10^3 + 2 \cdot 10^2 + 7 \cdot 10^1 + 9 \cdot 10^0$

3. Compute the following:

a. $2 \cdot 10^2 + 4 \cdot 10 + 7 \cdot 10^0$

$2 \cdot 100 + 4 \cdot 10 + 7 \cdot 1$

$= 200 + 40 + 7 = 240 + 7 = \boxed{247}$

b. $6 \cdot 10^4 + 3 \cdot 10^3 + 2 \cdot 10^2 + 9 \cdot 10 + 1 \cdot 10^0$

$6 \cdot 10000 + 3 \cdot 1000 + 2 \cdot 100 + 9 \cdot 10 + 1 \cdot 1$

$60{,}000 + 3{,}000 + 200 + 90 + 1$

$63{,}000 + 200 + 90 + 1$

$63{,}200 + 90 + 1 = 63{,}290 + 1 = \boxed{63{,}291}$

4. Evaluate the following numerical expressions using Order of Operations.

a. $6 \cdot 3 + 2^2 - 11 \cdot 4 \div 2$

$18 + 4 - 44 \div 2$

$22 - 22$

$\boxed{0}$

b. $3 \cdot 6 + 32 \div 4 - 2^3 \cdot 2$

$18 + 8 - 8 \cdot 2$

$26 - 16$

$\boxed{10}$

**SUMMARY (What I learned today)**