ADDING AND SUBTRACTING FRACTIONS 8

Name: Key Date:________ Period:_______

SECTION 8.5 COMMON DENOMINATORS AND MIXED NUMBERS

Big Idea: How do we add and subtract fractions with unlike denominators, including mixed numbers?

In the Exploration in Section 8.4, you discovered a rule for adding two fractions with unlike denominators:

$$\frac{1}{a}+\frac{1}{b}=\frac{1\cdot b}{a\cdot b}+\frac{1\cdot a}{b\cdot a}=\frac{b}{a\cdot b}+\frac{a}{a\cdot b}=\frac{b+a}{a\cdot b}$$

EXAMPLE 1

Assume p and q are primes. Compute the following two sums using the pattern from above. Notice that the first sum is a special case of the second, where p = 2 and q = 3.

a. $\frac{1}{6}+\frac{1}{9}$ b. $\frac{1}{pq}+\frac{1}{q^2}$

$2\cdot 3$ $3\cdot 3$

SOLUTION

Common Denominator Method:

As in the rule above, you can create a common denominator for each sum by multiplying the two given denominators:

$$\frac{1}{6}+\frac{1}{9}=\frac{1\cdot 9}{6\cdot 9}+\frac{1\cdot 6}{9\cdot 6}$$

$$=\frac{9+6}{6\cdot 9}=\frac{15}{54}$$

$$\frac{1}{pq}+\frac{1}{q^2}=\frac{1\cdot q^2}{pq\cdot q^2}+\frac{1\cdot pq}{q^2\cdot pq}$$

$$=\frac{q^2+pq}{pq^3}$$

No.

Are these fractions simplified? Do the numerator and denominator have any common factors? The numerator can be factored using the distributive property:

Yes.

$$\frac{15}{54}=\frac{3\cdot 5}{2\cdot 3\cdot 3\cdot 3}=\frac{5}{18}$$

common factor = 3

$$\frac{q^2+pq}{pq^3}=\frac{q(q+p)}{pq^3}=\frac{q+p}{pq^2}$$

common factor = q

Notice that this approach did not involve finding the LCD.

LCD Method:

Another approach is to first find the LCD of the fractions in each sum or, equivalently, the LCM of the denominators. Look at the prime factorizations of the denominators of the fractions: $6 = 2 \cdot 3$, $9 = 3^2$, and because p and q are primes, pq and q^2 are their own prime factorizations. Remember the rule for finding the LCM of two numbers from their prime factorizations: take the product of each prime raised to its larger exponent. So, the LCDs are LCM $(6, 9) = 2 \cdot 3^2$ and LCM $(pq, q^2) = pq^2$.

Now, in computing the sums $\frac{1}{6}+\frac{1}{9}$ and $\frac{1}{pq}+\frac{1}{q^2}$, multiply the numerator and denominator of each fraction by a factor that will make the denominator the LCD:

$$\frac{1}{6}+\frac{1}{9} = \frac{1}{2\cdot 3}+\frac{1}{3^2} = \frac{1\cdot 3}{(2\cdot 3)\cdot 3}+\frac{1\cdot 2}{3^2\cdot 2} = \frac{3+2}{2\cdot 3^2} = \frac{5}{18}$$

$$\frac{1}{pq}+\frac{1}{q^2} = \frac{1}{pq}+\frac{1}{q^2} = \frac{1\cdot q}{pq\cdot q}+\frac{1\cdot p}{q^2\cdot p} = \frac{q+p}{pq^2}$$

Notice that the final answer in each sum is already simplified.

Outline a procedure for adding or subtracting fractions with unlike denominators:

1. Find the LCD
2. Rewrite the fractions (equivalently) using the LCD
3. Compute the sum/difference

PROBLEM 1

Use the process above to compute the LCD for the following fractions and then compute the sum:

a. $\frac{1}{40}+\frac{1}{50}$

$\frac{1}{2\cdot 2\cdot 2\cdot 5}+\frac{1}{2\cdot 5\cdot 5}$

LCD $= 2\cdot 2\cdot 2\cdot 5\cdot 5 = 200$

$\frac{5}{200}+\frac{4}{200}$

$\boxed{\frac{9}{200}}$

b. $\frac{3}{8}+\frac{5}{12}$

LCD $= 24$

$\frac{9}{24}+\frac{10}{24}$

$\boxed{\frac{19}{24}}$

c. $\frac{7}{10}+\frac{4}{9}$

LCD $= 90$

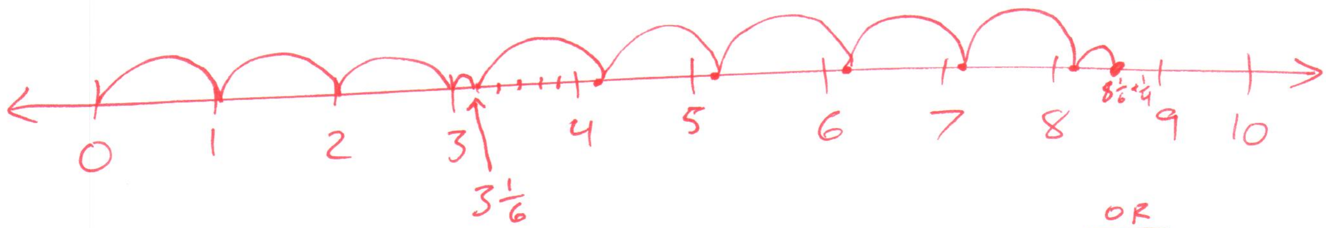
$\frac{63}{90}+\frac{40}{90}$

$\boxed{\frac{103}{90}}$

or $1\frac{13}{90}$

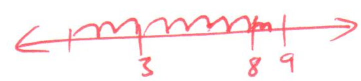
EXPLORATION 1

Silvia is baking six sheet cakes for a party. The recipe she is using calls for $3\frac{1}{6}$ pounds of refined sugar and $5\frac{1}{4}$ pounds of unrefined sugar. First, use the linear model to give an estimate of how much sugar Silvia needs. Then, compute how many pounds of sugar Silvia needs. Explain your process for both the estimation and the calculation. Can you use the same process to add other mixed numbers? Yes.



OR

Between 8 and 9 pounds of sugar.



$$3\tfrac{1}{6} + 5\tfrac{1}{4} = \left(3+\tfrac{1}{6}\right)+\left(5+\tfrac{1}{4}\right)$$

$$= (3+5)+\left(\tfrac{1}{6}+\tfrac{1}{4}\right) = 8 + \underbrace{\tfrac{1}{6}+\tfrac{1}{4}}_{\text{LCD is 12}} = 8+\tfrac{2}{12}+\tfrac{3}{12} = 8+\tfrac{5}{12}$$

$$= \boxed{8\tfrac{5}{12}\text{ lbs.}}$$

EXAMPLE 2

Compute the sum $6\frac{3}{5} + 3\frac{5}{7}$

$$\left(6+\tfrac{3}{5}\right)+\left(3+\tfrac{5}{7}\right) = (6+3)+\left(\tfrac{3}{5}+\tfrac{5}{7}\right) = 9+\left(\tfrac{3}{5}+\tfrac{5}{7}\right)$$

LCD is 35

$$= 9+\left(\tfrac{21}{35}+\tfrac{25}{35}\right) = 9+\left(\tfrac{46}{35}\right) = 9+\left(\tfrac{35+11}{35}\right)$$

$$= 9+\left(1\tfrac{11}{35}\right) = 9+\left(1+\tfrac{11}{35}\right) = 10+\tfrac{11}{35}$$

$$= \boxed{10\tfrac{11}{35}}$$

EXAMPLE 3

Compute the following differences:

a. $8\frac{4}{5} - 5\frac{3}{10}$

$= 8\frac{4}{5} - 5\frac{3}{10}$

$= \left(8 + \frac{4}{5}\right) - \left(5 + \frac{3}{10}\right)$

$= 8 - 5 + \frac{4}{5} - \frac{3}{10}$ LCD is 10

$= 3 + \frac{8}{10} - \frac{3}{10}$

$= 3 + \frac{5}{10}$

$= \boxed{3\frac{1}{2}}$

b. $6\frac{3}{5} - 3\frac{5}{7}$

$= \left(6 + \frac{3}{5}\right) - \left(3 + \frac{5}{7}\right)$

$= 6 + \frac{3}{5} - 3 - \frac{5}{7}$

$= 6 - 3 + \frac{3}{5} - \frac{5}{7}$ LCD is 35

$= 3 + \frac{21}{35} - \frac{25}{35}$

$= 3 + \frac{-4}{35} = 2 + 1 - \frac{4}{35}$

$= 2 + \frac{35}{35} - \frac{4}{35} = 2 + \frac{31}{35}$

$= \boxed{2\frac{31}{35}}$

PRACTICE EXERCISES

1. Compute the following sums and differences:

a. $\frac{7}{12} + \frac{3}{16}$ LCD = 48

$= \frac{28}{48} + \frac{9}{48}$

$= \boxed{\frac{37}{48}}$

b. $\frac{3}{8} + \frac{7}{10}$ LCD = 40

$= \frac{15}{40} + \frac{28}{40}$

$= \frac{43}{40} = \frac{40+3}{40}$

$= \boxed{1\frac{3}{40}}$

c. $\frac{11}{18} - \frac{5}{24}$

$2 \cdot 3 \cdot 3$ $2 \cdot 2 \cdot 2 \cdot 3$

LCD is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$ $= 72$

$= \frac{44}{72} - \frac{15}{72}$

$= \boxed{\frac{29}{72}}$

d. $\frac{3}{7} - \frac{2}{5}$ LCD is 35

$= \frac{15}{35} - \frac{14}{35}$

$= \boxed{\frac{1}{35}}$

e. $\frac{6}{m^2} + \frac{4}{mn}$ LCD is m^2n

$= \frac{6 \cdot n}{m^2 \cdot n} + \frac{4 \cdot m}{m \cdot mn}$

$= \boxed{\frac{6n + 4m}{m^2 n}}$

f. $\frac{1}{k} - \frac{5}{k^2}$

LCD is k^2

$= \frac{k}{k^2} - \frac{5}{k^2}$

$= \boxed{\frac{k-5}{k^2}}$

2. Compute the sums of mixed numbers using your method of choice.

a. $3\frac{1}{7} + 2\frac{1}{2}$

$3\frac{1}{7}$

$+2\frac{1}{2}$

$5 + \frac{1}{7} + \frac{1}{2}$

$= 5 + \frac{2}{14} + \frac{7}{14}$

$= \boxed{5\frac{9}{14}}$

b. $5\frac{3}{5} - 2\frac{2}{3}$

$5\frac{3}{5} - 2\frac{2}{3} \rightarrow 5\frac{9}{15} - 2\frac{10}{15} \rightarrow 4\frac{24}{15} - 2\frac{10}{15}$

$\boxed{2\frac{14}{15}}$

c. $13\frac{3}{4} - 2\frac{1}{6}$

$\left(13\frac{3}{4}\right) - \left(2\frac{1}{6}\right)$

$= 13 - 2 + \frac{3}{4} - \frac{1}{6}$

$= 11 + \frac{9}{12} - \frac{2}{12}$

$= 11 + \frac{7}{12}$

$= \boxed{11\frac{7}{12}}$

d. $1\frac{1}{13} + 6\frac{1}{39}$

$= 1 + \frac{1}{13} + 6 + \frac{1}{39}$ LCD is 39

$= 1 + \frac{3}{39} + 6 + \frac{1}{39}$

$= 7 + \frac{4}{39}$

$= \boxed{7\frac{4}{39}}$

e. $\frac{a}{b} + \frac{2}{a}$

$= \frac{a \cdot a}{b \cdot a} + \frac{2 \cdot b}{a \cdot b}$

$= \boxed{\frac{a^2 + 2b}{ab}}$

f. $4 + \frac{3}{t}$

$= \boxed{4\frac{3}{t}}$ or

$= \frac{4t}{1 \cdot t} + \frac{3}{t}$

$= \frac{4t + 3}{t}$

SUMMARY (What I learned today)