MULTIPLYING AND DIVIDING FRACTIONS 9

Name: *Key*   Date:_____   Period:_____

SECTION 9.1 MULTIPLICATION OF FRACTIONS

VOCABULARY

DEFINITION	EXAMPLE
Reciprocal: *if n is a non-zero number, the reciprocal (multiplicative inverse) is $\frac{1}{n}$. Also, $\frac{1}{n} \cdot n = 1$*	$\frac{1}{5} \cdot 5 = 1$ *$\frac{1}{5}$ and 5 are reciprocals of each other.*

**Big Idea:** How do you multiply fractions?

EXAMPLE 1

Jane has $\frac{1}{2}$ yard of ribbon and cuts $\frac{1}{3}$ of its length. To do this, she finds out what is $\frac{1}{3}$ of $\frac{1}{2}$. How long is the piece of ribbon that she cut? Draw a number line to illustrate this length. Show this with an area model as well.

$\frac{1}{6}$ of a yard

$\frac{1}{6}$
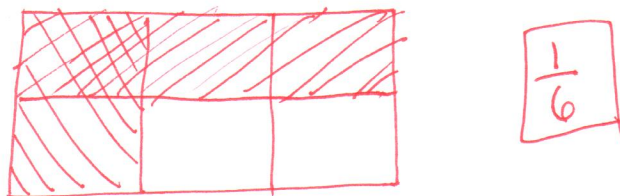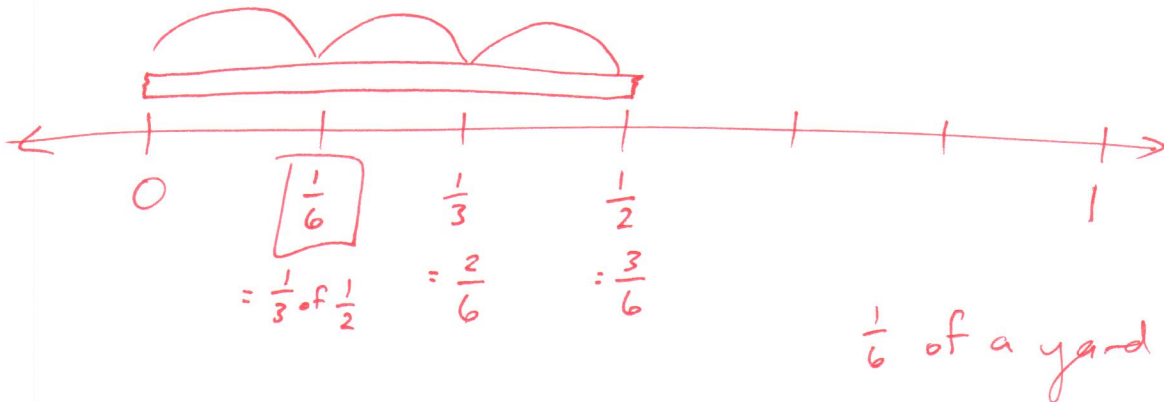
EXLORATION 1

a. Translate $\frac{1}{2}$ of $\frac{1}{5}$ into a multiplication problem and draw the corresponding picture to find the product.
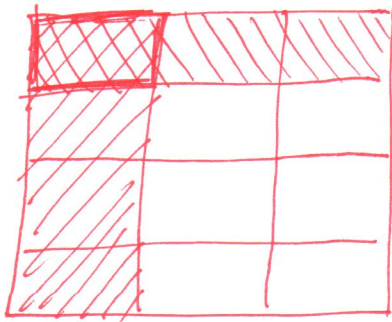
$$\frac{1}{2} \cdot \frac{1}{5} = \boxed{\frac{1}{10}}$$
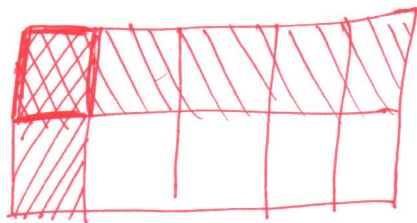
b. Translate $\frac{1}{4}$ of $\frac{1}{3}$ into a multiplication problem and draw the corresponding picture to find the product.

$$\frac{1}{4} \cdot \frac{1}{3} = \boxed{\frac{1}{12}}$$

c. Predict $\frac{1}{6}$ of $\frac{1}{7}$ without drawing a model.

$$\frac{1}{6} \cdot \frac{1}{7} = \frac{1 \cdot 1}{6 \cdot 7} = \boxed{\frac{1}{42}}$$
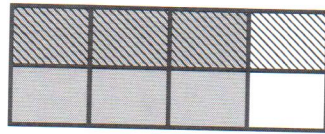
d.   Explain why multiplication of unit fractions is commutative. That is,

$$\text{why is } \left(\frac{1}{m}\right)\left(\frac{1}{n}\right) = \left(\frac{1}{n}\right)\left(\frac{1}{m}\right) ?$$

$mn = nm$ for integers.

so, $\dfrac{1}{m} \cdot \dfrac{1}{n} = \dfrac{1 \cdot 1}{m \cdot n} = \dfrac{1 \cdot 1}{n \cdot m} = \dfrac{1}{n} \cdot \dfrac{1}{m}$

PROBLEM 2

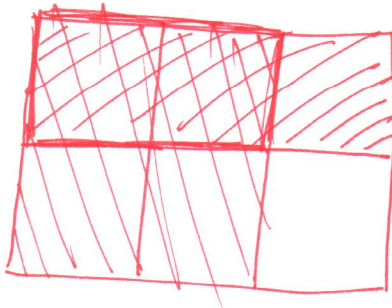What is $\frac{1}{2}$ of $\frac{3}{4}$?

$\dfrac{3}{8}$

EXPLORATION 2

Use the area model to illustrate and find the following answers.
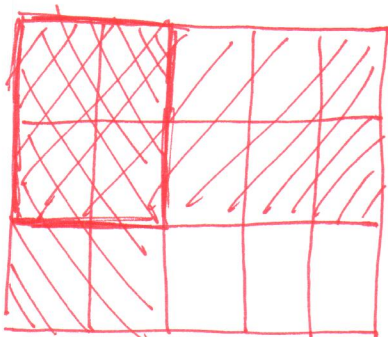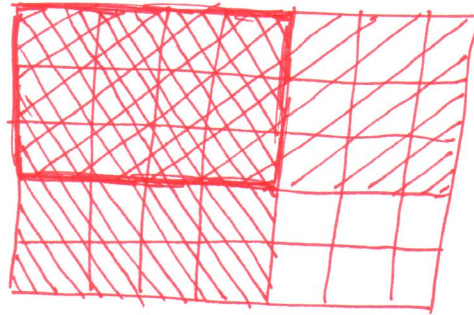
   a.   What is $\frac{1}{2}$ of $\frac{2}{3}$?

$\dfrac{1}{2} \cdot \dfrac{2}{3} = \dfrac{2}{6} = \boxed{\dfrac{1}{3}}$

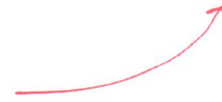   b.   What is $\frac{2}{5}$ of $\frac{2}{3}$?

$\dfrac{2}{5} \cdot \dfrac{2}{3} = \boxed{\dfrac{4}{15}}$

c. What is $\frac{3}{5}$ of $\frac{4}{7}$?

$$\frac{3}{5} \cdot \frac{4}{7} = \frac{12}{35}$$

d. Predict $\frac{3}{4}$ of $\frac{5}{7}$ without drawing a model.

$$\frac{3}{4} \cdot \frac{5}{7} = \frac{3 \cdot 5}{4 \cdot 7} = \frac{15}{28}$$

e. What do you notice about the product of two proper fractions?

The product is less than each of the factors.

f. What is the product of the fractions $\frac{a}{b}$ and $\frac{c}{d}$, where a, b, c and d are integers and each of b and d is not zero?

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

PROBLEM 3

Multiply the following fractions:

a. $\frac{1}{2} \cdot \frac{3}{4} = \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8}$

c. $\frac{3}{5} \cdot \frac{5}{3} = \frac{3 \cdot 5}{5 \cdot 3}$
$= \frac{15}{15} = 1$

e. $\frac{5}{9} \cdot \frac{3}{4} = \frac{5 \cdot 3}{9 \cdot 4}$
$= \frac{15}{36} = \frac{5}{12}$

b. $\frac{3}{5} \cdot \frac{2}{7} = \frac{3 \cdot 2}{57} = \frac{6}{35}$

d. $\frac{3}{4} \cdot \frac{4}{3} = \frac{3 \cdot 4}{4 \cdot 3} = \frac{12}{12} = 1$

EXAMPLE 2

Compute the product $\frac{21}{32} \cdot \frac{16}{35}$ = $\frac{336}{1120}$ = $\frac{3}{10}$

or

$$\frac{21 \cdot 16}{32 \cdot 35} = \frac{21 \cdot 16}{35 \cdot 32} = \frac{21}{35} \cdot \frac{16}{32} = \frac{3}{5} \cdot \frac{1}{2} = \boxed{\frac{3}{10}}$$

simplify

PROBLEM 4

Compute each of the following and simplify as needed.

a. $5 \cdot \frac{3}{5}$ = $\frac{5}{1} \cdot \frac{3}{5}$ = $\frac{15}{5}$ = $\boxed{3}$

b. $\frac{36}{49} \cdot \frac{13}{15}$ = $\frac{36 \cdot 13}{49 \cdot 15}$ = $\frac{36 \cdot 13}{15 \cdot 49}$

= $\frac{12 \cdot 13}{5 \cdot 49}$ = $\boxed{\frac{156}{245}}$

c. $\frac{12}{17} \cdot \frac{5}{24}$ = $\frac{12 \cdot 5}{17 \cdot 24}$ = $\frac{12 \cdot 5}{24 \cdot 17}$

= $\frac{12}{24} \cdot \frac{5}{17}$ = $\frac{1}{2} \cdot \frac{5}{17}$

= $\frac{5}{34}$

d. $\frac{24}{25} \cdot \frac{15}{36}$ = $\frac{24 \cdot 15}{25 \cdot 36}$ = $\frac{24 \cdot 15}{36 \cdot 25}$

$\frac{24}{36} \cdot \frac{15}{25}$ = $\frac{2}{3} \cdot \frac{3}{5}$ = $\frac{2 \cdot 3}{3 \cdot 5}$

$\frac{3 \cdot 2}{3 \cdot 5}$ = $\boxed{\frac{2}{5}}$

EXPLORATION 3

What fraction can be multiplied by $\frac{2}{3}$ to get 1? In other words, what times $\frac{2}{3}$ equals 1? Explain your answer.

$\frac{3}{2}$

because $\frac{2}{3} \cdot \frac{3}{2} = 1$

Remember that the **reciprocal** of a number is the number that, when multiplied by the original number, equals 1. What is the reciprocal of $\frac{3}{4}$? Verify that the product of $\frac{3}{4}$ and its reciprocal is 1.

$\frac{4}{3}$

$\frac{4}{3} \cdot \frac{3}{4} = 1$

Make a conjecture about the reciprocal of any fraction $\frac{a}{b}$.

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

For example, the reciprocal of $\frac{x}{y}$ is $\frac{y}{x}$. This makes sense because $\frac{x}{y} \cdot \frac{y}{x} = \frac{xy}{yx} = \frac{xy}{xy} = 1$.

You found that the product of $\frac{2}{3}$ and $\frac{3}{2}$ equals 1, and the fractions are reciprocals of each other. Notice that $\frac{3}{2}$ is a fraction larger than 1. In general, if a positive number is less than 1, then its reciprocal is greater than 1.

Rule 9.3: RECIPROCAL OF A FRACTION

In general, the **multiplicative inverse** or **reciprocal** of $\frac{x}{y}$ is the fraction $\frac{y}{x}$ since

$$\frac{x}{y} \cdot \frac{y}{x} = \frac{x \cdot y}{y \cdot x} = \frac{xy}{xy} = 1.$$

EXAMPLE 3

Lisa has 24 books in her library, one third of which are hardback books.

a. How many of her library books are hardback?

$$\frac{1}{3} \cdot 24 = \frac{1}{3} \cdot \frac{24}{1} = \frac{24}{3} = \boxed{8}$$

b. How many of her books are not hardback?

$$24 - 8 = \boxed{16} \qquad \text{OR} \qquad \frac{2}{3} \cdot 24 = \frac{2 \cdot 24}{3} = \frac{48}{3} = \boxed{16}$$

PROBLEM 5

Compute each of the following and simplify if needed.

a. $12 \cdot \frac{1}{4}$

$$\frac{12}{1} \cdot \frac{1}{4} = \frac{12}{4} = \boxed{3}$$

b. $12 \cdot \frac{1}{3}$

$$\frac{12}{1} \cdot \frac{1}{3} = \frac{12}{3} = \boxed{4}$$

c. $\dfrac{4}{5} \cdot \dfrac{1}{3}$

$\dfrac{4 \cdot 1}{5 \cdot 3} = \boxed{\dfrac{4}{15}}$

d. $\dfrac{3}{7} \cdot 35$

$\dfrac{3}{7} \cdot \dfrac{35}{1} = \dfrac{3 \cdot 35}{7 \cdot 1} = \dfrac{35 \cdot 3}{7 \cdot 1}$

$= \dfrac{35}{7} \cdot \dfrac{3}{1} = 5 \cdot 3 = \boxed{15}$

OR $\dfrac{3}{7} \cdot \dfrac{35}{1} = \dfrac{105}{7} = \boxed{15}$

PROBLEM 6

Find the products of each of the following problems. Write your answer as an improper fraction in simplest form and as a simplified mixed number.

a. $4\dfrac{1}{2} \cdot 3\dfrac{2}{3}$

$= \dfrac{9}{2} \cdot \dfrac{11}{3} = \dfrac{99}{6} = \boxed{\dfrac{33}{2} = 16\dfrac{1}{2}}$

c. $2\dfrac{3}{4} \cdot 4\dfrac{2}{3}$

$= \left(\dfrac{2 \cdot 4 + 3}{4}\right)\left(\dfrac{4 \cdot 3 + 2}{3}\right)$

$= \dfrac{11}{4} \cdot \dfrac{14}{3} = \dfrac{154}{12}$

$= \boxed{\dfrac{77}{6}} = \dfrac{72+5}{6} = \boxed{12\dfrac{5}{6}}$

b. $3\dfrac{2}{5} \cdot 1\dfrac{1}{3}$

$= \left(\dfrac{3 \cdot 5}{5} + \dfrac{2}{5}\right)\left(\dfrac{3}{3} + \dfrac{1}{3}\right)$

$= \dfrac{17}{5} \cdot \dfrac{4}{3} = \boxed{\dfrac{68}{15} = 4\dfrac{8}{15}}$

d. $3\dfrac{4}{5} \cdot 7\dfrac{1}{3}$

$= \dfrac{3 \cdot 5 + 4}{5} \cdot \dfrac{7 \cdot 3 + 1}{3}$

$= \dfrac{19}{5} \cdot \dfrac{22}{3} = \boxed{\dfrac{418}{15}}$

$= \dfrac{405+13}{15} = \boxed{27\dfrac{13}{15}}$

SUMMARY (What I learned today)