MULTIPLYING AND DIVIDING FRACTIONS   9

Name: _____Key_____ Date:_____ Period:_____

SECTION 9.4 FRACTIONS AND ALTERNATIVES

Big Idea: How do we apply fractions, decimals, and percents to real-life questions?

EXPLORATION 1

Order the numbers below from least to greatest and identify which of the numbers are equal to each other. Explain your reasoning.

$$\frac{1}{3}, \quad 30\%, \quad 0.33, \quad 0.3, \quad \frac{3}{10}, \quad \frac{1}{3}\%$$

$33.\overline{3}\%$ $\qquad$ 33% $\quad$ 30% $\quad$ 30% $\quad$ $0.00\overline{3}\%$

$$0.00\overline{3}\% < 30\% = 30\% = 30\% < 33\% < 33.\overline{3}\%$$

$$\boxed{\frac{1}{3}\% < 30\% = 0.3 = \frac{3}{10} < 0.33 < \frac{1}{3}}$$

PROBLEM 1

There are normally 36 students in Terry's class. However, due to a flu epidemic 25% of the class is absent today. How many students are absent? Explain how you got your answer.

25% is $\frac{1}{4} = \frac{25}{100}$

$$\left(\frac{1}{4}\right)(36) = \frac{36}{4} = 36 \div 4 = \boxed{9}$$

Explanations may vary.

EXPLORATION 2

a. Roger needs 75% of a 160-centimeter rope. Decide which of the following equivalent representations would be the easiest for him to use to determine the amount of rope he needs: 75%, 0.75, or $\frac{3}{4}$.

$$160 \cdot 0.75$$

b. If he has 200 centimeters and wants to use $\frac{1}{5}$ of it? Decide which of the following equivalent representations would be easiest for him to use: 20%, 0.20, or $\frac{1}{5}$.

$$200 \cdot \frac{1}{5} = \frac{200}{5}$$

c. If Roger wanted $\frac{1}{6}$ of the 200-centimeter rope, which method, fraction or decimal, would be better?

Fraction to divide 200 by 6 $200 \cdot \frac{1}{6} = \frac{200}{6}$

Decimal to multiply.

d. In the problems above, what determines which method is better?

One way to decide is to look at the fraction & determine if it is "friendly"

PROBLEM 2

Solve each of the following pairs of equivalent equations. In each pair, which form was easier to solve? Explain.

a. $\frac{1}{4}x = 3$ and $0.25x = 3$

$$\left(4\right)\left(\frac{1}{4}x\right) = \left(3\right)\left(4\right)$$

$$x = 12$$

$$0.25x = 3$$

$$\frac{0.25x}{0.25} = \frac{3}{0.25}$$

$$x = 12$$

b. $\frac{1}{6}x = 4$ and $(0.1666\ldots)x = 4$

$$6 \cdot \frac{1}{6}x = 4 \cdot 6$$

$$x = 24$$

$$(0.1666\ldots)x = 4$$

$$\frac{(0.1666\ldots)x}{0.1666\ldots} = \frac{4}{0.1666\ldots}$$

$$x = 24$$

c. $\dfrac{4}{5}x = 7$ and $0.8x = 7$

$$\dfrac{5}{4}\left(\dfrac{4}{5}x\right) = (7)\dfrac{5}{4}$$

$$\dfrac{5\cdot 4}{4\cdot 5}x = \dfrac{7\cdot 5}{4}$$

$$x = \dfrac{35}{4}$$

$$x = 8\dfrac{3}{4}$$

$$\dfrac{0.8x = 7}{(0.8)} \quad \dfrac{7}{(0.8)}$$

$$x = 7 \div 0.8$$

$$x = 8.75$$

$$7 \div 0.8$$
$$= 70 \div 8$$

$$\begin{array}{r} 8.75 \\ 8\overline{)70.00} \\ -64 \\ \hline 60 \\ -56 \\ \hline 40 \\ 40 \end{array}$$

EXAMPLE 1

There are 18 green marbles in Julia's bag. Two-thirds of the marbles in her bag are green. How many marbles does she have in the bag? Justify each step.

$$18 = \dfrac{2}{3}M$$

$$\left(\dfrac{3}{2}\right)(18) = \left(\dfrac{3}{2}\right)\left(\dfrac{2}{3}M\right)$$

$$\dfrac{54}{2} = M$$

$$\boxed{27 = M}$$

EXAMPLE 2

Amy has some pens in her desk. Nathan starts out with twice as many pens as Amy. Nathan then gives $\dfrac{1}{3}$ of his pens to his sister Lisa. Lisa received 18 pens from Nathan. How many pens must Amy have in her desk?

$$N = 2A$$

$$L = \dfrac{1}{3}N = 18$$

Lisa received 18 pens
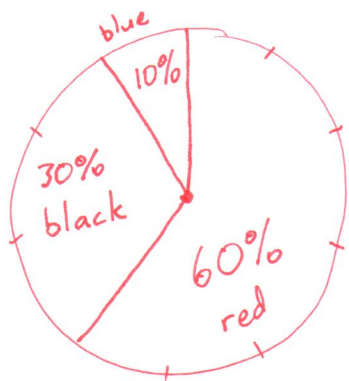
$$18 = \dfrac{1}{3}(2A) = \dfrac{2}{3}A$$

$$\dfrac{3}{2}(18) = \dfrac{3}{2}\left(\dfrac{2}{3}A\right)$$

$$\boxed{27 = A}$$

EXAMPLE 3

In a random survey to find people's favorite car color, 60% of the people surveyed liked red, 10% liked blue, and 30% liked black.

a.   Represent this data with a pie graph.

b.   If 12 people in the survey liked red, how many people total were surveyed? Solve numerically, and also by using your pie graph.

$T = $ total people surveyed

$12 = .6\,T$

$\dfrac{12}{.6} = \dfrac{.6\,T}{.6}$

$20 = T$

$\dfrac{12}{T} = \dfrac{60}{100}$

$T\dfrac{12}{T} = \dfrac{60\,T}{100}$

$10(12) = \left(\dfrac{6T}{10}\right)10$

$120 = 6T$

$\boxed{20 = T}$

c.   Based on this survey, in a group of 1000 people, about how many might be expected to like red?

60% of 1000

$(.6)(1000) = \boxed{600 \text{ people}}$

PRACTICE EXERCISES

1. Solve each of the following pairs of equivalent equations:

a. $\dfrac{3}{4}x = 15$ and $(0.75)x = 15$

$$\frac{4}{3}\left(\frac{3}{4}x\right) = (15)\frac{4}{3}$$

$$x = \frac{15 \cdot 4}{3}$$

$$x = 5 \cdot 4 \quad \boxed{x = 20}$$

$$\frac{0.75\,x}{0.75} = \frac{15}{0.75}$$

$$\boxed{x = 20}$$

$$15 \div 0.75$$
$$= 1500 \div 75$$

$$\begin{array}{r} 20 \\ 75\overline{)1500} \\ \underline{-150} \\ 0 \end{array}$$

b. $\dfrac{5}{8}y = \dfrac{9}{2}$ and $(0.625)y = 4.5$

$$\frac{8}{5}\left(\frac{5}{8}y\right) = \frac{8}{5}\left(\frac{9}{2}\right)$$

$$y = \frac{72}{10}$$

$$y = \frac{36}{5} \quad \boxed{y = 7\frac{1}{5}}$$

$$\frac{0.625\,y}{0.625} = \frac{4.5}{0.625}$$

$$\boxed{y = 7.2}$$

$$4.5 \div 0.625$$
$$4500 \div 625$$

$$\begin{array}{r} 7.2 \\ 625\overline{)4500.0} \\ \underline{-4375} \\ 1250 \\ \underline{-1250} \\ 0 \end{array}$$

2. Jade has $\dfrac{1}{5}$ of a gallon of paint left over and spills $\dfrac{2}{3}$ of it. How much paint is left in the container?

$$\frac{2}{3} \text{ of } \frac{1}{5} \qquad \frac{2}{3} \cdot \frac{1}{5} = \frac{2 \cdot 1}{3 \cdot 5} = \boxed{\frac{2}{15} \text{ gallons}}$$

3. A fish tank is $\dfrac{4}{5}$ full when it starts to leak, and losses 75% of the water. How much of the tank did it lose?

$$75\% \text{ of } \frac{4}{5} \qquad \text{is} \qquad \frac{3}{4} \text{ of } \frac{4}{5}$$

$$\frac{3}{4} \cdot \frac{4}{5} = \frac{3 \cdot 4}{4 \cdot 5} = \frac{3}{5} \text{ of the tank, or}$$

$$60\% \text{ of the tank.}$$

SUMMARY (What I learned today)