RATES, RATIOS AND PROPORTIONS 10

Name: Key Date: Period:

SECTION 10.1 RATES AND RATIOS

VOCABULARY

DEFINITION	EXAMPLE
Ratio: a division comparison of 2 quantities	$\frac{7}{10}$ 7 to 10 7:10
Equivalent ratios: ratios which simplify to equivalent fractions	7 to 10 AND 70:100
Rate: a division comparison of 2 quantities, with 2 different units	$\frac{48 \text{ dollars}}{6 \text{ hours}}$
Unit rate: a rate per one unit.	60 miles per hour or 60 $\frac{\text{miles}}{\text{hour}}$

Big Idea: What are rates and ratios and how do we use them?

What are three different ways to write or express ratios?

3:5 3 to 5 $\frac{3}{5}$

EXPLORATION 1

Ms. Eusebi's math class has 12 boys and 18 girls. Write the following comparisons as ratios: (methods may vary.)

a. The number of boys to the number of girls 12 to 18 or 2:3 or $\frac{2}{3}$

b. The number of girls to the number of students 18 to 30 or 3:5 or $\frac{3}{5}$

c. The number of students to the number of girls 30 to 18 or 5:3 or $\frac{5}{3}$

d. The number of boys to the number of students

12 to 30 or 2:5 or $\frac{2}{5}$

e. The number of students to the number of boys

30 to 12 or 5:2 or $\frac{5}{2}$

EXPLORATION 2

Juan drove 150 miles in 3 hours and used 5 gallons of gasoline. Make as many rates as you can from these quantities and their units as possible. Explain what each unit fraction means.

$\frac{150 \text{ miles}}{3 \text{ hours}} = 50$ miles per hour

$\frac{3 \text{ hours}}{150 \text{ miles}} = \frac{1}{50}$ hours per mile

$\frac{3 \text{ hours}}{5 \text{ gallons}} = \frac{3}{5}$ hours per gallon

$\frac{5 \text{ gallons}}{3 \text{ hours}} = \frac{5}{3}$ gallons per hour

$\frac{150 \text{ miles}}{5 \text{ gallons}} = 30$ miles per gallon

$\frac{5 \text{ gallons}}{150 \text{ miles}} = \frac{1}{30}$ gallons per mile

PROBLEM 1

Karla rode her bike $2\frac{1}{2}$ hours and traveled 20 miles. What was her average rate, or speed? Approximately how far did she travel in the first hour and a half?

$\text{speed} = \text{rate} = \frac{\text{distance}}{\text{time}} = \frac{20 \text{ miles}}{2.5 \text{ hours}} = 8 \frac{\text{miles}}{\text{hour}} =$ 8 miles per hour

$\text{distance} = \text{rate} \cdot \text{time} = (8 \text{ miles per hour})(1.5 \text{ hours})$

$= (8 \frac{\text{miles}}{\text{hour}})(1.5 \text{ hours}) =$ 12 miles

EXAMPLE 1

Kirsten's bakery uses 12 cups of flour to make 3 cakes. How many cups of flour will she need to bake 7 cakes for a customer? Use the table below.

12 ÷ 3

Cups of flour	4	8	12	16	20	24	28
Number of cakes	1	2	3	4	5	6	7

How can we use the unit rate to solve this problem?

Unit rate: $\frac{12 \text{ cups}}{3 \text{ cakes}} = 4 \frac{\text{cups}}{\text{cake}}$

Multiply the unit rate by the number of cakes:

$4 \cdot 7 =$ 28 cups of flour

EXPLORATION 3

Amanda works as a computer consultant at the Lennox Company and earns $612 for working 36 hours. If she charges a fixed amount per hour, how much will she earn working 5 hours? 10 hours? 15 hours? Use unit rate to solve and show your work in table form.

Unit rate: $\frac{\$612}{36 \text{ hours}} = 17 \frac{\text{dollars}}{\text{hour}}$

# hours	$ earned
5 hours	$(5 \text{ hours})\left(17 \frac{\text{dollars}}{\text{hour}}\right) =$ $85
10 hours	$(10 \text{ hours})\left(17 \frac{\text{dollars}}{\text{hour}}\right) =$ $170
15 hours	$(15 \text{ hours})\left(17 \frac{\text{dollars}}{\text{hour}}\right) =$ $255

PROBLEM 2

Karen and Karla both jog every day. Karen jogs an average of 1,800 meters in 40 minutes. Karla jogs an average of 1,500 meters in 30 minutes.

a. Who jogs faster?

Karen: $\frac{1800\text{ m}}{40\text{ min}} = 45$ meters per minute

Karla: $\frac{1500\text{ m}}{30\text{ min}} = 50$ meters per minute

Karla is faster

b. On average, how far does Karen jog in 12 minutes?

$(12\text{ minutes})\left(45\frac{\text{meters}}{\text{minute}}\right) = 12 \cdot 45\text{ meters} = 540\text{ m}$

c. Maintaining the pace, how long does it take Karla to jog 600 meters?

$d = rt \qquad (600\text{ m}) = \left(50\frac{\text{m}}{\text{min}}\right)t$

$(600\text{m}) \cdot \left(\frac{1\text{ min}}{50\text{ m}}\right) = \left(\frac{50\text{m}}{1\text{ min}}\right)\left(\frac{1\text{ min}}{50\text{ m}}\right)t$ $\qquad t = 12\text{ min}$

How does the formula for rate relate to unit rates?

$d = rt$ usually describes "rate" (r) in unit rates

Examples: mph = miles per hour = $\frac{\text{miles}}{\text{hour}}$

feet per second = $\frac{\text{ft}}{\text{second}}$

PROBLEM 3

Sandy went canoeing for 3 hours and traveled 12 miles down river. How fast was she traveling in miles per hour?

$\frac{12\text{ miles}}{3\text{ hours}} = \frac{12}{3}\text{ miles per hour}$

$= 4\text{ miles per hour}$

At the same rate, how far will she travel in 5 hours?

$d = rt$

$(4\text{ miles per hour})(5\text{ hours}) = \left(4\frac{\text{miles}}{\text{hr}}\right)(5\text{ hr})$

$= 20\text{ miles}$

Some typical equivalents with the corresponding conversion rates equal to 1 are:

3 feet = 1 yard	$\frac{3 \text{ ft}}{1 \text{ yd}} = \frac{1 \text{ yd}}{3 \text{ ft}} = 1$
1760 yards = 1 mile	$\frac{1760 \text{ yds}}{1 \text{ mile}} = \frac{1 \text{ mile}}{1760 \text{ yds}} = 1$
1 mile = 5280 feet	$\frac{5280 \text{ ft}}{1 \text{ mile}} = \frac{1 \text{ mile}}{5280 \text{ ft}} = 1$
10 millimeters = 1 centimeter	$\frac{10 \text{ mm}}{1 \text{ cm}} = \frac{1 \text{ cm}}{10 \text{ mm}} = 1$
60 minutes = 1 hour	$\frac{60 \text{ min}}{1 \text{ hr}} = \frac{1 \text{ hr}}{60 \text{ min}} = 1$
8 pints = 1 gallon	$\frac{8 \text{ pints}}{1 \text{ gal}} = \frac{1 \text{ gal}}{8 \text{ pints}} = 1$
1 gallon = 128 ounces	$\frac{128 \text{ oz}}{1 \text{ gal}} = \frac{1 \text{ gal}}{128 \text{ oz}} = 1$

EXAMPLE 2

a. Convert 4 miles to yards

$(4 \text{ miles})\left(\frac{1760 \text{ yds}}{1 \text{ mile}}\right) = 7040 \text{ yards}$

b. Convert $3\frac{1}{2}$ hours to minutes

$(3.5 \text{ hr})\left(\frac{60 \text{ min}}{1 \text{ hr}}\right) = 210 \text{ minutes}$

c. Convert 640 ounces to gallons $(640 \text{ oz.})\left(\frac{1 \text{ gal}}{128 \text{ oz.}}\right) = 5 \text{ gallons}$

EXAMPLE 3

Susan has $5\frac{1}{3}$ yards of cloth. How many feet of cloth is this? How many inches of cloth does she have?

$= \frac{16}{3}$ of cloth

$\left(\frac{16}{3} \text{ yds}\right)\left(\frac{3 \text{ ft}}{1 \text{ yd}}\right) = 16 \text{ ft}$

$(16 \text{ ft})\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) = 192 \text{ in}$

OR $\left(\frac{16}{3} \text{ yd}\right)\left(\frac{36 \text{ in}}{1 \text{ yd}}\right) = 192 \text{ in}$

PRACTICE EXERCISES

1. Dierdre has 2 notebooks, 5 pencils, and 3 pens in her backpack.

   a. What is the ratio of notebooks to pencils?

   2 : 5

   b. What is the ratio of pencils to pens?

   5 : 3

   c. What is the ratio of pens to notebooks?

   3 : 2

2. Hugo, the dog, can walk 18 blocks in 3 hours. How many blocks will he travel at a constant rate in 1 hour? How far can he travel in $2\frac{1}{4}$ hours? $2\frac{1}{4} = \frac{9}{4}$

$\frac{18 \text{ blocks}}{3 \text{ hours}} = \boxed{6 \frac{\text{blocks}}{\text{hr}}}$

$\left(6 \frac{\text{blocks}}{\text{hr}}\right)\left(\frac{9}{4} \text{ hr}\right) = \frac{54}{4}$ blocks $= \boxed{\frac{27}{2} \text{ blocks}}$

13.5 blocks

3. Taylor went to the store to buy cereal. She found one cereal that 16 ounces for \$3.89, another that was 12 ounces for \$2.79, and a third that was 20 ounces for \$5.32. Which is the best deal? What are the unit rates of each cereal?

$\frac{\$3.89}{16 \text{ oz.}} = \$0.24$ per oz.

$\frac{\$2.79}{12 \text{ oz.}} = \$0.23$ per oz. ← Best Deal

$\frac{\$5.32}{20 \text{ oz.}} = \$0.27$ per oz.

4. Use dimensional analysis to convert the following:

a. $5\frac{1}{2}$ cups = $\frac{11}{32}$ gallons

b. 3.8 km = 2.356 miles

c. $16\frac{3}{4}$ yards = 603 inches

a. $5\frac{1}{2} \text{ cups} = \left(\frac{11}{2} \text{ cups}\right)\left(\frac{1 \text{ gallon}}{16 \text{ cups}}\right) = \frac{11}{32}$ gallons

b. $(3.8 \text{ km})\left(\frac{0.62 \text{ mi}}{1 \text{ km}}\right) = 2.356$ miles

c. $16\frac{3}{4} \text{ yd} = \left(\frac{67}{4} \text{ yd}\right)\left(\frac{36 \text{ in}}{1 \text{ yd}}\right) = 603$ in

5. Sam is driving his car at 45 mph.

a. At this rate, how many feet does he travel in one hour?

$\frac{45 \text{ mi}}{1 \text{ hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = 237{,}600 \text{ ft in one hr}$

b. How many feet does he travel in one second?

$\frac{237{,}600 \text{ ft}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ seconds}} = 66 \text{ ft per second}$

c. Create a table showing Sam's distance traveled in 15 minute increments from zero to 2 hrs.

time (min)	0	15	30	45	60	75	90	105	120
distance (miles)	0	11.25	22.5	33.75	45	56.25	67.5	78.75	90

SUMMARY (What I learned today)