# RATES, RATIOS AND PROPORTIONS 10

Name: Key Date:________ Period:______

## SECTION 10.2 RATES OF CHANGE AND LINEAR FUNCTIONS

**Big Idea:** How do rates of change relate to linear functions?

**ACTIVITY**
Working in groups of three, mark off a distance of 10 ft. on the floor with tape. Each square on the ground is equal to 1 ft.

**Step 1**: Have one team member walk slowly for 10 ft. Have another team member time the walk and the third member record the time.

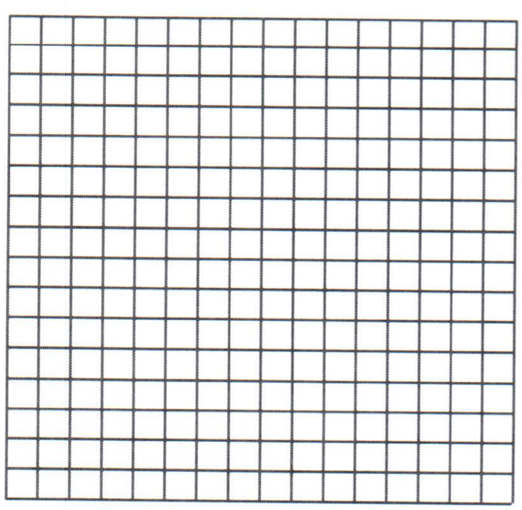
**Step 2**: Record the time for the same member walking at a regular pace for 10 ft.

**Step 3**: Record the time for the same member walking at a quick pace for 10 ft.

**Step 4**: Use the times to estimate the time for each walker to walk 20 feet, 30 feet, and 40 feet, assuming the walker neither speeds up nor slows down when walking farther.

**Step 5**: Make a graph of the rates for the three walkers.

Answers will vary.



| # of feet | Seconds for the Slow Rate | Seconds for the Medium Rate | Seconds for the Fast Rate |
|---|---|---|---|
| 10 | | | |
| 20 | | | |
| 30 | | | |
| 40 | | | |

1. What quantities are changing in each graph? What remains the same?

Time and rate change, while the distance is the same for each.

2. Determine the rate of change using quantities that you identified. Locate the rate of change in your graph.

↳ Answers will vary.

$d = rt$

$\text{rate} = \frac{\text{distance}}{\text{time}}$ which is the slope.

3. Determine the unit rate of change in your graph (you can round to the nearest tenth place)

Answers will vary.
Unit should be $\frac{\text{ft}}{\text{second}}$.

**EXPLORATION 1**

A restaurant makes and sells a famous dish that contains rice and beans. The ratio of rice to beans in its secret recipe is 1:2, the ratio of beans to rice is 2:1

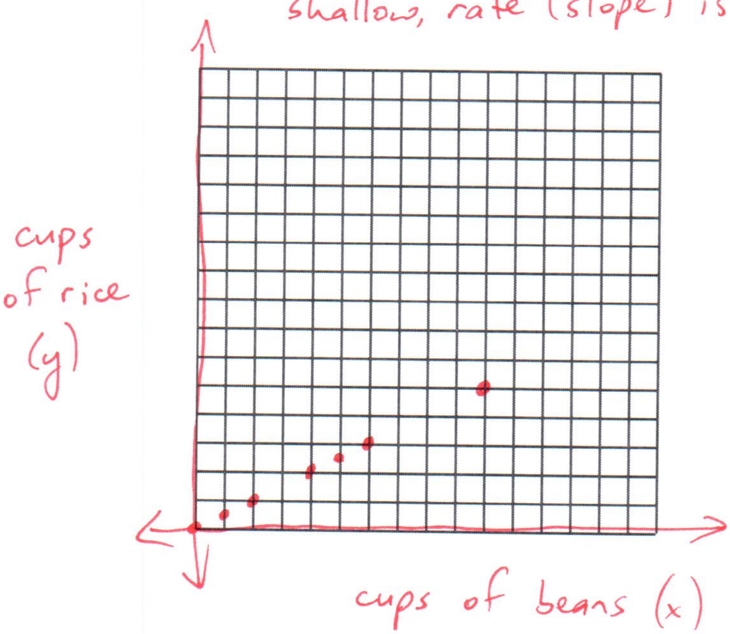
**Step 1**: Make a table of possible amounts of rice and beans that the chef uses to make the dish. How did you organize the first column of numbers? What is the first number in the beans column?

Answers may vary.

| Cups of Beans ($x$) | Cups of Rice ($y$) | $\frac{Rice}{Beans} = \left(\frac{y}{x}\right)$ |
|---|---|---|
| 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| 2 | 1 | $\frac{1}{2}$ |
| 4 | 2 | $\frac{1}{2}$ |
| 6 | 3 | $\frac{1}{2}$ |
| 0 | 0 | n/a |
| 5 | 2.5 | $\frac{1}{2}$ |
| 10 | 5 | $\frac{1}{2}$ |

**Step 2**: Use the numbers in the table as coordinates of points. Make a graph using the data points. For each point, the number of cups of beans is the x-coordinate, and the number of cups of rice is the y-coordinate. Describe the graph.

shallow, rate (slope) is $\frac{1}{2}$.



**Step 3**: For each of the rows in the table, what is the ratio of rice to beans? How is the second column of amounts of rice changing?

1 to 2. Amount of rice is changing based on amount of beans. Increase of 1 cup of beans corresponds with increase of $\frac{1}{2}$ cup of rice.

**Step 4**: Could you start with a smaller amount of beans? Pick any two smaller amounts of beans and find the corresponding amounts of rice.

Yes. Choices of amounts may vary.

Example: $\frac{1}{2}$ cup beans : $\frac{1}{4}$ cup rice

$\frac{1}{3}$ cup beans : $\frac{1}{6}$ cup rice

**Step 5**: The graph is a straight line. Define $R(x)$ as the number of cups of rice needed for x cups of beans in the recipe. Write a rule for this linear function $R$. What is the ratio of rice to beans?

$R =$ $\frac{1}{2}(x)$

rice to beans → $\frac{1 \text{ cup rice}}{2 \text{ cups beans}}$

**Step 6**: Use the rule for $R$ to compute the amount of rice needed to 3 ½ cups of beans. Use the graph to confirm that your answer makes sense.

$R = \frac{1}{2}\left(3\frac{1}{2}\right)$

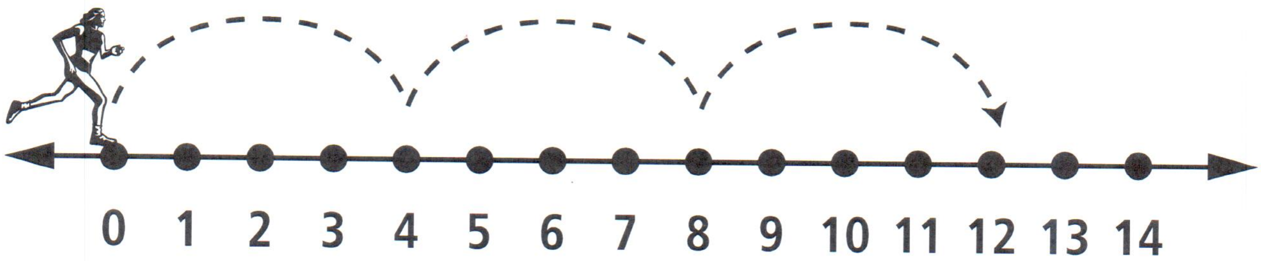
$= \left(\frac{1}{2}\right)\left(\frac{7}{2}\right)$

$= \boxed{\frac{7}{4} \text{ cups of rice}}$

OR $1\frac{3}{4}$ cups of rice

**EXAMPLE 1**

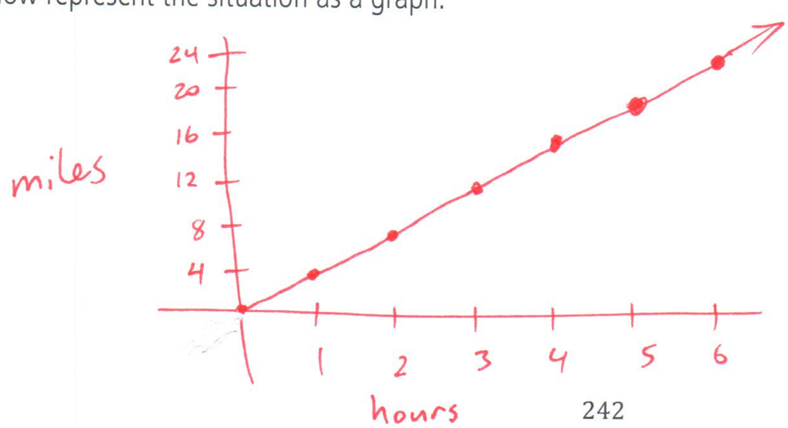
Priscilla takes a long walk on the weekend, walking at a steady rate of 4 miles per hour. Using the formula $d = rt$, if $r = 4$, the distance $d$ depends on the amount of time $t$ that Priscilla walks. This process is like the frog-jumping model on the number line from Section 4.1.




From the picture, make the following table

| Time ($t$) | Distance ($d$) | Rate $\left(r = \frac{d}{t}\right)$ |
|---|---|---|
| 0 | 0 | — |
| 1 | 4 | 4 |
| 2 | 8 | 4 |
| 3 | 12 | 4 |
| 4 | 16 | 4 |
| 5 | 20 | 4 |
| 6 | 24 | 4 |

Now represent the situation as a graph.



## PROBLEM 1

In the graph below, $d$ represents the distance traveled by a bicycle rider and $t$ represents the number of hours ridden.

a. What is the rate in miles per hour?

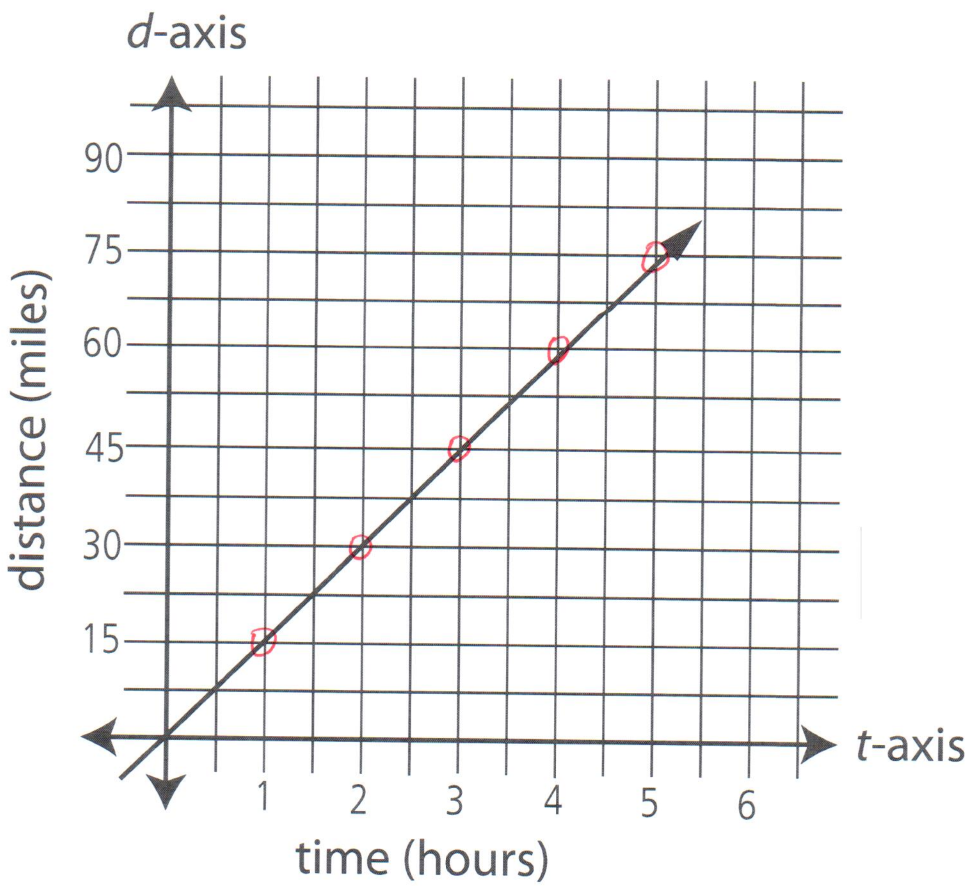
$15 \frac{\text{miles}}{\text{hr}}$

b. Use this rate to write a formula for $d$ in terms of $t$.

$d = rt$ $\quad d = 15t$

c. How did you figure out the rate of change?

Use a table, or look at the rate (slope) of the graph.



## EXAMPLE 2

Jane bought 4 drinks for \$9.

a. What is her unit rate of cost per drink?

$\frac{9}{4} = 2\frac{1}{4} = \$2.25$

b. Based on this, fill out the given table.

c. Write a function that gives the total cost of drinks in terms of the number of drinks, $x$, purchased.

$C(x) = 2.25x$

d. Does the cost function have a constant rate of change? If so, what is it? Where is it in the equation? What do you notice about the third column?

| Number of drinks purchased ($x$) | Total cost of drinks $y = C(x)$ | $y \div x$ |
|---|---|---|
| 1 | 2.25 | 2.25 |
| 2 | 4.50 | 2.25 |
| 3 | 6.75 | 2.25 |
| 4 | 9.00 | 2.25 |
| 5 | 11.25 | 2.25 |
| 6 | 13.50 | 2.25 |
| 7 | 15.75 | 2.25 |

Yes. 2.25, multiplied by x.

The 3rd column is the rate (and is constant)

e. Is there another way to solve this problem without filling out a table?

Yes, algebraically or by graphing.

## PROBLEM 2

Suppose a linear function has a constant rate of change of 2.5. Write an equation for this function and make a table with at least 5 points on this function.

$$\frac{y}{x} = 2.5$$

$$x\left(\frac{y}{x}\right) = x(2.5)$$

$$y = x(2.5)$$

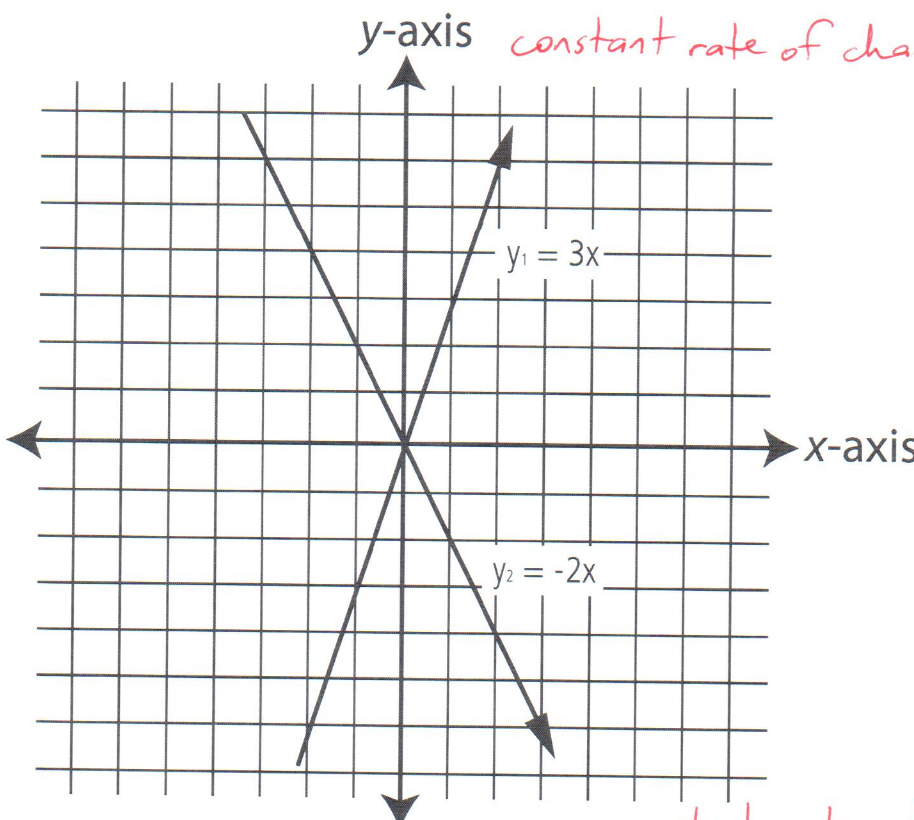
$$\boxed{y = 2.5x}$$

| x | y |
|---|---|
| 1 | 2.5 |
| 2 | 5 |
| 3 | 7.5 |
| 4 | 10 |
| 5 | 12.5 |

## PROBLEM 3

Find the constant rate of change for the linear functions represented by their graphs on the coordinate system below:

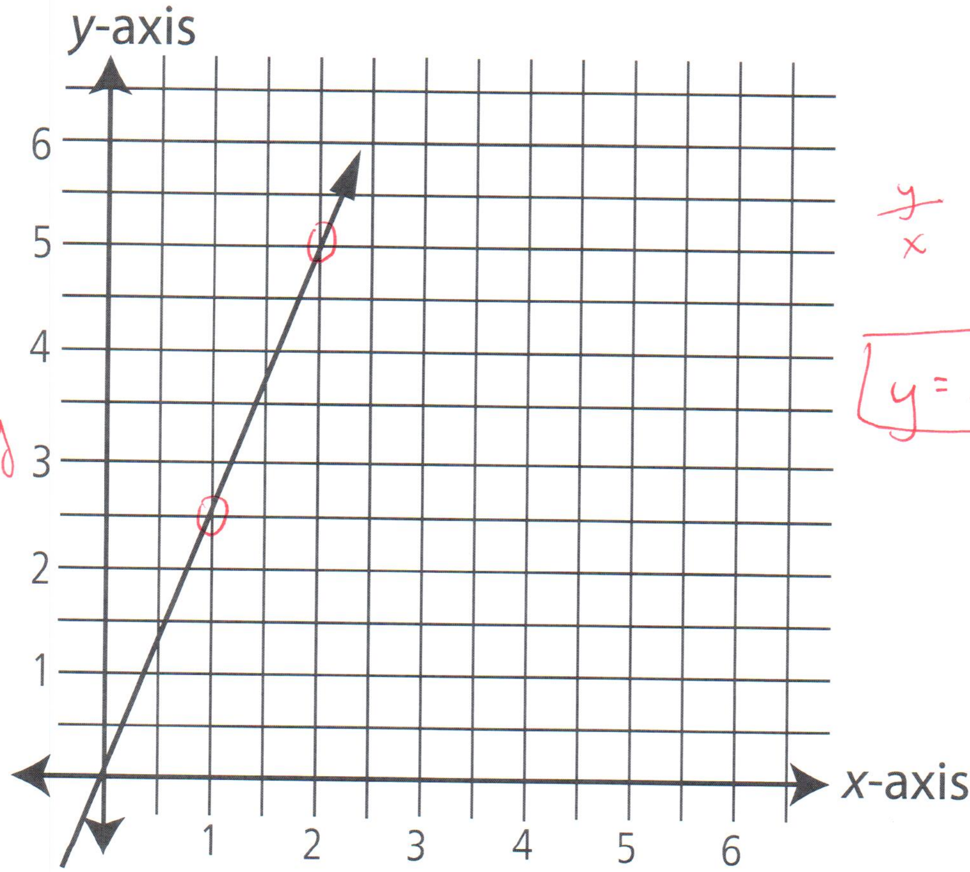
constant rate of change = 3




constant rate of change is −2

**PROBLEM 4**

In the graph below, y represents the number of teaspoons of honey used to make *x* number of cupcakes. What is the ratio of teaspoons of honey to number of cupcakes? Use this ratio to write a formula for *y* in terms of *x*.




honey

cupcakes

$\frac{y}{x} = \frac{2.5}{1}$ or $\frac{5}{2}$

$\boxed{y = 2.5x}$

**PRACTICE EXERCISES**

1. Write an equation and make a table with five points for a linear function with both a constant rate of change and proportionality of $2\frac{1}{2}$.

| x | y |
|---|---|
| 0 | 0 |
| 1 | 2.5 |
| 2 | 5 |
| 3 | 7.5 |
| 4 | 10 |
| 5 | 1.25 |

Chosen points may vary.

2. The table below is from a linear function. What is the constant rate of change for this function? What is the linear function? Fill in the table for the ratio of $y$ to $x$.

$-6$ $y = -6x$

| $x$ | $y$ | $\frac{y}{x}$ |
|---|---|---|
| -2 | 12 | $\frac{12}{-2} = -6$ |
| -1 | 6 | $\frac{6}{-1} = -6$ |
| 0 | 0 | $\frac{0}{0}$ $\boxed{N/A}$ |
| 1 | -6 | $\frac{-6}{1} = -6$ |
| 2 | -12 | $\frac{-12}{2} = -6$ |

3. Sunshine brand sunflower seeds have 20 mg of fat and 8g of protein per serving.

a. Write a function P($x$) for the number of grams of protein if $x$ grams of fat are eaten. What is P($x$)?

P(x) to x is 8 to 20

$$x\left(\frac{P(x)}{x}\right) = \left(\frac{8}{20}\right)x$$

$\boxed{P(x) = 0.4x}$ or $P(x) = \frac{2}{5}x$

b. Write a function F($x$) for the grams of fat if $x$ grams of protein are eaten. What is F($x$)?

$$x \cdot \frac{F(x)}{x} = \frac{20}{8} \cdot x$$

$F(x) = \frac{5}{2} \cdot x$ or $\boxed{F(x) = 2.5x}$

## SUMMARY (What I learned today)