RATES, RATIOS AND PROPORTIONS 10

Name: Key Date:_____ Period:_____

SECTION 10.3 PROPORTIONS

VOCABULARY

DEFINITION	EXAMPLE
Proportion: an equation of ratios in the form $\frac{a}{b}=\frac{c}{d}$, where b and d are not equal to zero	$\frac{3}{6}=\frac{1}{2}$ $\frac{x}{y}=\frac{3}{4}$

Big Idea: How do we use proportional thinking to solve problems?

$\frac{x \text{ miles}}{2 \text{ in}}=\frac{50 \text{ miles}}{1 \text{ in}}$

EXPLORATION 1

When you look at a map of Texas, you know that the actual state is much larger than the map. For example, one inch can represent 50 miles according to a scale designation on the map legend. That means that the ratio of the map distance to the actual distance is 1 inch to 50 miles. This ratio is written 1:50 or $\frac{1}{50}$.

Using this information, what actual distance does 2 inches represent? This time, writing the information as a ratio of actual distance to the map distance, the fraction is $\frac{x}{2}$, where x is the actual distance in miles, represented by 2 inches on the map. Combine the two ratios in the equation $\frac{x \text{ miles}}{2 \text{ inches}}=\frac{50 \text{ miles}}{1 \text{ inch}}$

Can you solve this equation for x? Explain. Yes, multiply both sides by 2 in.

$2 \text{ inches}\left(\frac{x \text{ miles}}{2 \text{ inches}}\right)=\left(\frac{50 \text{ miles}}{1 \text{ inch}}\right)2 \text{ inches}$

$x \text{ miles}=\frac{100 \text{ miles} \cdot \text{inches}}{1 \text{ inch}}$

$x \text{ miles}=100 \text{ miles}$

$x=100$

EXAMPLE 1

Four water bottles cost $6. How much will it cost to buy 10 of the same water bottles?

a. Set up a proportion using x, the cost to buy 10 bottles, that would allow you to solve this problem:

$$\frac{\$6}{4 \text{ bottles}} = \frac{\$x}{10 \text{ bottles}}$$

b. Write two other ways to set up this proportion:

$$\frac{\$x}{\$6} = \frac{10 \text{ bottles}}{4 \text{ bottles}} \quad \text{or} \quad \frac{4 \text{ bottles}}{\$6} = \frac{10 \text{ bottles}}{\$x}$$

$$\text{or} \quad \frac{\$6}{\$x} = \frac{4 \text{ bottles}}{10 \text{ bottles}}$$

c. Solve for x using any of your proportions. Does it matter which proportion you use if it is set up correctly?

It does not matter.

$$10 \text{ bottles}\left(\frac{\$6}{4 \text{ bottles}}\right) = \left(\frac{\$x}{10 \text{ bottles}}\right)10 \text{ bottles}$$

$$\frac{\$60}{4} = \frac{\$x}{1}$$

$$\$15 = \$x$$

$$\boxed{x = 15}$$

EXAMPLE 2

Set up the following problem using a proportion. Three bags of chips cost $2.79. How much do 7 bags of chips cost? Write the four different proportions that are possible and solve one of them.

$$\frac{3 \text{ bags}}{7 \text{ bags}} = \frac{\$2.79}{\$C}$$

$$\frac{3 \text{ bags}}{\$2.79} = \frac{7 \text{ bags}}{\$C}$$

$$\frac{\$C}{7 \text{ bags}} = \frac{\$2.79}{3 \text{ bags}}$$

$$\frac{\$C}{\$2.75} = \frac{7 \text{ bags}}{3 \text{ bags}}$$

C = cost of 7 bags

$$7 \text{ bag}\left(\frac{\$C}{7 \text{ bags}}\right) = \left(\frac{\$2.79}{3 \text{ bags}}\right)7 \text{ bags}$$

$$\$C = \boxed{\$6.51}$$

EXAMPLE 3

Marla estimates her party guests will consume an average of a pint of punch each, so she will need 28 pints of punch. She has a family recipe that makes one gallon of punch. How many gallons of punch does she need for the party?

Tabular Method:

pints	4	8	12	16	20	24	28	32
gallons	0.5	1	1.5	2	2.5	3	3.5	4

She needs 3.5 gallons

Unit Rate Method:

$\frac{1 \text{ gallon}}{8 \text{ pints}} = \frac{1}{8}$ gallons per pint $= \frac{1}{8} \frac{\text{gallons}}{\text{pint}}$

$\frac{1}{8} \cdot 28 = 3.5$ or $0.125 \cdot 28 = 3.5$

3.5 gallons

Proportion Method:

1 gallon = 4 quarts

1 quart = 2 pints

↓

1 gallon = 8 pints

$$\frac{28 \text{ pints}}{1} \left(\frac{g \text{ gallons}}{28 \text{ pints}} \right) = \left(\frac{1 \text{ gallon}}{8 \text{ pints}} \right) \frac{28 \text{ pints}}{1}$$

g gallons $= \frac{28}{8}$ gallons

$g = 3.5$

EXAMPLE 4

A colony of leafcutter ants cuts up 4 leaves in 7 minutes. How many leaves does the colony cut in an hour?

Tabular Method:

Time in minutes	Number of leaves cut
0	0
7	4
14	8
21	12
28	16
35	20
42	24
49	28
56	32
63	36

They cut between 32 and 36 leaves in an hour.

Unit Rate Method: 1 hour = 60 minutes

Ants cut $\frac{4 \text{ leaves}}{7 \text{ minutes}} = \frac{4}{7}$ leaves per minute

$\frac{4}{7} \cdot 60 = \frac{240}{7} = \boxed{34\frac{2}{7} \text{ leaves}}$ in an hour

Proportion Method:

$$60 \text{ min}\left(\frac{x \text{ leaves}}{60 \text{ minutes}}\right) = \left(\frac{4 \text{ leaves}}{7 \text{ minutes}}\right) 60 \text{ min}$$

$$x \text{ leaves} = \frac{240}{7} \text{ leaves}$$

$$\boxed{x = 34\frac{2}{7}}$$

EXAMPLE 5

Leo runs $\frac{1}{4}$ of a mile in 3 minutes. Assuming he continues to run at the same rate, how many miles will he run in 45 minutes? Solve using both tabular and proportions method.

Tabular Method:

Time (in minutes)	Distance (in miles)
0	0
3	$\frac{1}{4} = 0.25$
6	$\frac{1}{2} = 0.5$
12	$1 = 1$
15	$1\frac{1}{4} = 1.25$
30	$2\frac{1}{2} = 2.5$
45	$3\frac{3}{4} = 3.75$

3.75 miles in 45 minutes

Proportion Method:

$$45 \text{ min}\left(\frac{x \text{ miles}}{45 \text{ minutes}}\right) = \left(\frac{\frac{1}{4} \text{ miles}}{3 \text{ minutes}}\right) 45 \text{ min}$$

$$x \text{ miles} = \left(\frac{1}{4} \cdot \frac{1}{3}\right)(45) \text{ miles}$$

$$x \text{ miles} = \frac{45}{12} \text{ miles}$$

$$x = 3\frac{9}{12}$$

$$x = 3\frac{3}{4}$$

$3\frac{3}{4}$ miles in 45 minutes

EXAMPLE 6

Lucinda is studying prairie dog populations in Colorado. She captures and tags 15 prairie dogs and then releases them back into the wild. Two weeks later she captures 35 prairie dogs and discovers 3 are tagged. What is the approximate population of prairie dogs in the region? Solve using proportions.

$$\frac{x \text{ total}}{15 \text{ tagged}} = \frac{35 \text{ total}}{3 \text{ tagged}}$$

$$15 \text{ tagged}\left(\frac{x \text{ total}}{15 \text{ tagged}}\right) = \left(\frac{35 \text{ total}}{3 \text{ tagged}}\right) 15 \text{ tagged}$$

$$x \text{ total} = \frac{35 \cdot 15}{3} \text{ total}$$

$x = 175$

The approximate population is 175 prairie dogs.

EXPLORATION 2

From Math Explorer, December 1999, vol. 2.3

A globe is approximately 30 cm in diameter. Using that measurement, calculate the scaled size for the measurements indicated in the table.

Object	Actual size (miles)	Scaled size (cm)
Earth diameter	8,000	30
Top of the atmosphere	100	$100 \cdot \frac{0.00375}{1} = 0.375$
Space shuttle orbit height	200	$2 \cdot 0.00375 = 0.75$
Height of satellite	18,000	$18{,}000 \cdot 0.00375 = 67.5$
Moon diameter	2,100	$2100 \cdot 0.00375 = 7.875$
Distance from Earth to Moon	240,000	$240{,}000 \cdot 0.00375 = 900$

$\frac{30\text{ cm}}{8000\text{ mi}} = 0.00375 \frac{\text{cm}}{\text{mi}}$

PRACTICE EXERCISES

1. Solve the following equations:

a. $\frac{14}{42} = \frac{x}{6} \rightarrow \frac{2}{6} = \frac{x}{6}$ $x = 2$

b. $\frac{x}{15} = \frac{3}{5} \rightarrow 15\left(\frac{x}{15}\right) = \left(\frac{3}{5}\right)15 \rightarrow x = \frac{45}{4}$

$x = 9$

c. $\frac{16}{x} = \frac{4}{9} \rightarrow x\left(\frac{16}{x}\right) = \left(\frac{4}{9}\right)x \rightarrow 16 = \frac{4}{9}x \rightarrow 9(16) = \left(\frac{4}{9}\right)9x$

$144 = 4x \rightarrow \frac{144}{4} = \frac{4x}{4} \rightarrow 36 = x$

2. Dried cherries are sold at $70 for 5 lbs.

a. At this price, how much does 3 lbs. cost? Set up a proportion to solve.

$$3\text{ lbs}\left(\frac{\$70}{5\text{ lbs}}\right)=\left(\frac{\$x}{3\text{ lbs}}\right)3\text{ lbs}$$

$$\frac{\$210}{5}=\$x$$

$$\$42=\$x$$

3 lbs cost **$42**

b. What is the unit price per pound?

$$\frac{\$70}{5\text{ lbs}}=\frac{\$x}{1\text{ lb.}}$$

$$\$14=\$x$$

One pound is $14.

$14 per pound

3. Thomas is studying Mexican free-tail bats in caves. He catches and tags 80 bats from a cave near Austin, Texas. After a month he catches another 150 and discovers 6 are tagged. What is the approximate population of bats in the cave?

$$80\text{ tagged}\left(\frac{P\text{ bats total}}{80\text{ tagged}}\right)=\left(\frac{150\text{ total}}{6\text{ tagged}}\right)80\text{ tagged}$$

$$P\text{ total}=$$ **2000 bats total in the cave**

SUMMARY (What I learned today)