RATES, RATIOS AND PROPORTIONS 10

Name: Key Date: _____ Period: _____

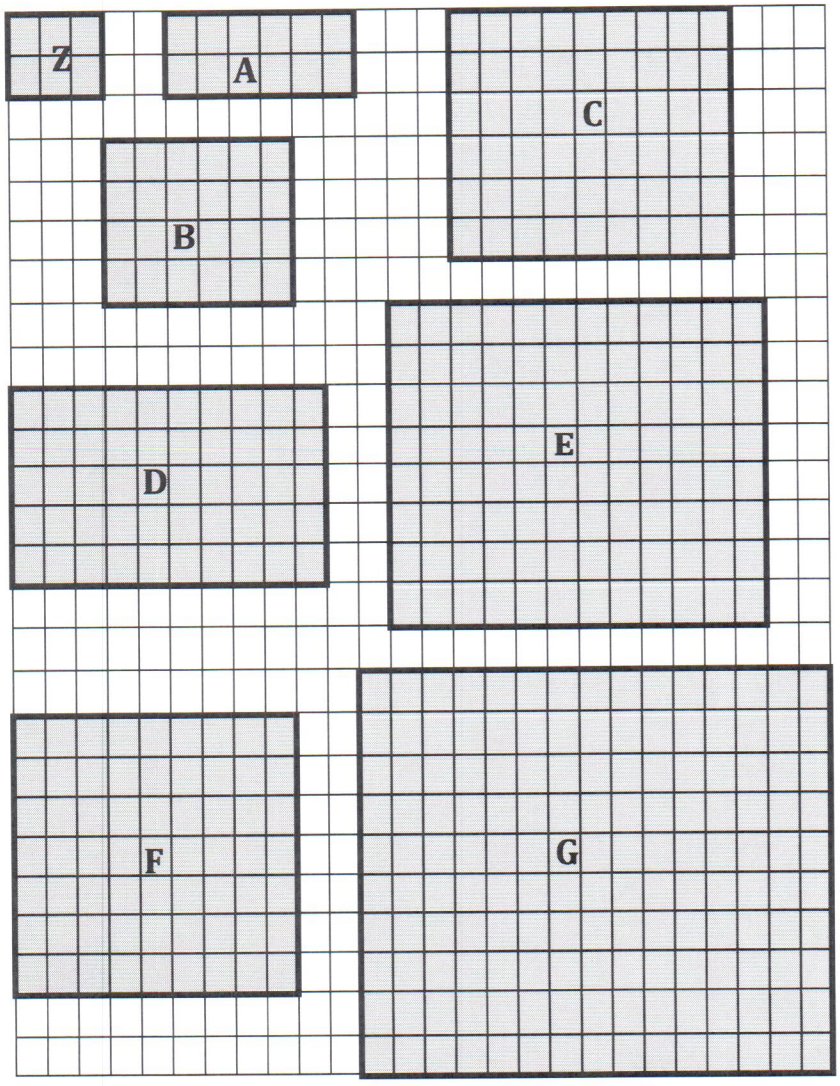
SECTION 10.5 SCALING

VOCABULARY

DEFINITION	EXAMPLE
Scale factor: If rectangle A has dimensions b and h, and B has dimensions kb and kh, k is a scale factor (also called constant of proportionality)	2 [A 3] k=2 4 [B 6]

Big Idea: How can we use rates and ratios to scale figures?

EXPLORATION 1: Discuss the different ways the following rectangles can be compared.



a. What are the attributes of a rectangle?

- length
- width
- perimeter
- area

b. How would you group these rectangles together if you had to sort them? Which would you pair with which?

answers will vary.

Ex: Z, B, C, E, G have the same ratio of side lengths

c. Which rectangles have exactly the same shape as rectangle Z? Explain.

B, C, E, and G have the same shape but different sizes, their sides have a 2 to 3 ratio

Rectangle	Height	Base
Z	3	2
A	2	6
B	4	6
C	6	9
D	5	10
E	8	12
F	7	9
G	10	15

d. Complete the following table. What do you notice about rectangles B, C, E and G?

heights are multiples of 3

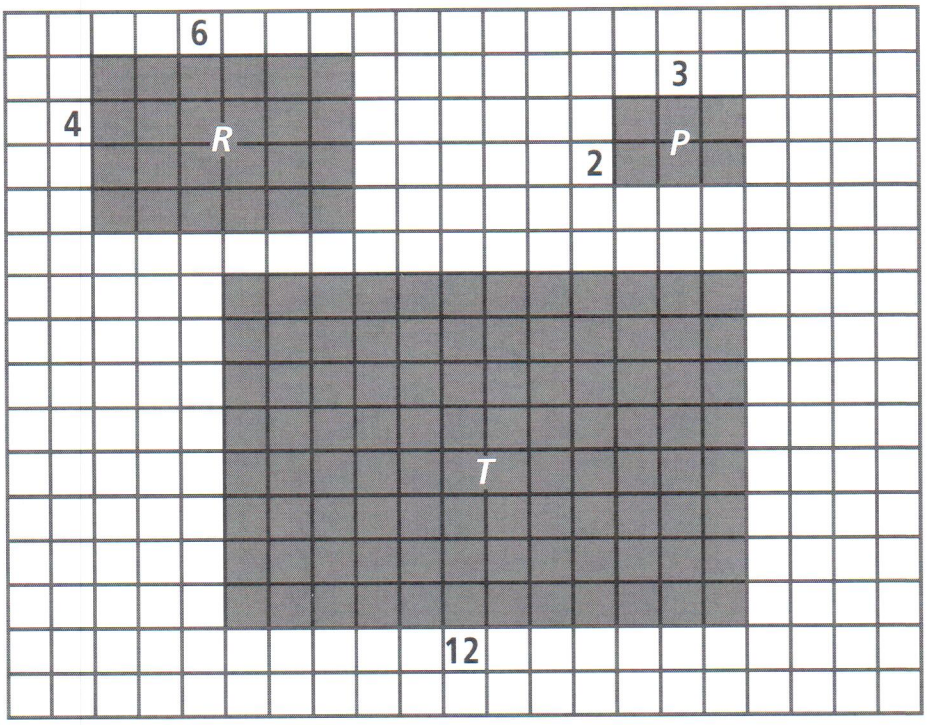
bases are multiples of 2

e. What do rectangles B, C, E & G have in common?

ratio of sides

PROBLEM 1

Suppose we start with the rectangle P below. What is the scale factor from P to R? From R to T? From T to T? From R to P? From P to T? From T to P?



P to R 2

R to T 2

T to T 1

R to P $\frac{1}{2}$

P to T 4

T to P $\frac{1}{4}$

EXPLORATION 2

Make a 4 by 8 rectangle on a blank grid paper and label it R. Create 5 more rectangles that have the same shape but different size from R and label them A, B, C, D and E. What is the scale factor of each new rectangle in relation to the rectangle R?

a. Complete the table below and complete the data. Answers will vary, examples below.

Rectangle	Length	Width	Scale Factor	Perimeter	Area
R	4	8	1	24	32
A	5	10	$\frac{5}{4}$	30	50
B	8	16	2	48	128
C					
D					
E					
F	2	4	$\frac{1}{2}$	12	8

b. Did you create a rectangle that is smaller than R? If not, make one now and label it F.

example in table

c. What is the scale factor from R to F? Make a conjecture about scale factors less than 1.

Example: $\frac{1}{2}$

Scale factors less than 1 occur when the figure "becomes" smaller

d. Make a rectangle the same shape as R that has length 6 units long. Label it G. What is its scale factor?

4 [R] 8 6 [G] 12 scale factor = $\frac{3}{2}$ $\frac{6}{4} = \frac{4k}{4}$ $k = \frac{3}{2}$

e. Make a new rectangle using a scale factor of $\frac{1}{4}$. Label it H. What are the dimensions of this new rectangle? How can you check to see if the scale factor is $\frac{1}{4}$?

1 [H] 2 1 by 2 $\frac{1}{\frac{1}{4}} = 4$ $\frac{2}{\frac{1}{4}} = 8$ so $\frac{1}{4}$ is the scale factor

f. Make a rectangle with the same shape as R that has length 3 units long. Label it J. What is the scale factor for this rectangle?

3 [J] 6 scale factor = $\frac{3}{4}$ $\frac{3}{4} = \frac{4k}{4}$ $k = \frac{3}{4}$

EXAMPLE 1

Sue is making a dessert from a recipe that will serve 3 people. The recipe uses 5 tablespoons of chocolate, and 2 cups of milk.

a. Sue wants to serve six people. How much milk will she need?

3 people to 6 people

scale factor = 2

(2 cups of milk)(2) =

4 cups of milk

b. Sue decides to invite four more people. How much milk will she need now? What is the scale factor between her final recipe and the original?

Scale factor = x

$\frac{3x}{3} = \frac{10}{3}$

$x = \frac{10}{3}$

$\frac{10}{3}$(2 cups of milk) = $\frac{20}{3}$ cups of milk

or $6\frac{2}{3}$ cups of milk

c. On another occasion, Sue uses 7 tablespoons of chocolate. How much milk should she use this time?

x is the scale factor

$\frac{5x}{5} = \frac{7}{5}$

$x = \frac{7}{5}$

(2 cups of milk)$\left(\frac{7}{5}\right)$ = $\frac{14}{5}$ cups of milk

or $2\frac{4}{5}$ cups of milk

PROBLEM 2

Make a 5 units by 9 units rectangle on grid paper and label it Rectangle M.

a. Construct a new rectangle N using a scale factor of 0.6. Label the new dimensions of rectangle N.

3 by 5.4 (N: 3 × 5.4; 5 · 0.6 and 9 · 0.6)

b. Construct another rectangle P using a scale factor of 1.5. Label the new dimensions of rectangle P.

7.5 by 13.5 (P: 7.5 × 13.5; 5 · 1.5 and 9 · 1.5)

PROBLEM 3

Suppose Rectangle A is a 6 by 9 rectangle. Draw Rectangle B so that the scale factor from A to B is $\frac{1}{3}$.

a. What is the scale factor from B to A? 3 (B: 2 × 3; $\frac{2k}{2} = \frac{6}{2}$, $k = \frac{6}{2} = 3$)

b. Draw a Rectangle C so that the scale factor from B to C is 2. (C: 4 × 6)

c. What is the scale factor from A to C? $\frac{2}{3}$ ($\frac{6k}{6} = \frac{4}{6}$, $k = \frac{4}{6} = \frac{2}{3}$)

d. What is the scale factor from C to A? $\frac{3}{2}$ ($\frac{4k}{4} = \frac{6}{4}$, $k = \frac{6}{4} = \frac{3}{2}$)

e. What do you notice about the scale factors from A to B and B to A? From A to C and C to A?

reciprocals; also reciprocals

PROBLEM 4

a. Draw a rectangle that is 3 inches by 4 inches. On a map with a scale factor of 1 inch = 20 miles, how big an area does the rectangle represent?

$3 \text{ in} \cdot \frac{20 \text{ miles}}{1 \text{ inch}} = 60 \text{ miles}$

$4 \text{ inches} \cdot \frac{20 \text{ miles}}{1 \text{ inch}} = 80 \text{ miles}$

$\text{Area} = (60)(80) = 4800 \text{ miles}^2$

b. What are the dimensions of a rectangle that represents an area 50 miles wide and 80 miles long on the map? What is the area of the scaled rectangle? What is the area represented by the map?

Dimensions on map: $50 \text{ miles} \cdot \frac{1 \text{ inch}}{20 \text{ miles}} = 2.5 \text{ inches}$ $80 \text{ mi} \cdot \frac{1 \text{ in}}{20 \text{ mi}} = 4 \text{ inches}$

Area scaled rectangle: $(2.5)(4) = 10 \text{ in}^2$

Area represented: $(50 \text{ miles})(80 \text{ miles}) = 4000 \text{ miles}^2$

or $10 \text{ in}^2 \left(\frac{20 \text{ mi}}{1 \text{ in}}\right)\left(\frac{20 \text{ mi}}{1 \text{ in}}\right) = 4000 \text{ miles}^2$

PRACTICE EXERCISES

1. Mrs. Freese is using a projector to enlarge a picture of her schools mascot to paint on her classroom wall. If the original picture's dimensions are 4 in by 6 in, and she wants to make it 10 ft by 15 ft, what scale factor will she use?

$(4 \text{ in})k = (10 \text{ ft})$

$k = \frac{10 \text{ ft}}{4 \text{ in}} = \frac{5 \text{ ft}}{2 \text{ in}}$

$5 \text{ ft} = (5 \cdot 12 \text{ in}) = 60 \text{ in}$

$k = \frac{60 \text{ in}}{2 \text{ in}} = 30 = k$

2. Decrease the length and the width of rectangle M by $\frac{1}{2}$ to form rectangle N.

   a. What is the scale factor from M to N?

   $\frac{1}{2}$

   b. What is the scale factor from N to M?

   2 (the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$)

SUMMARY (What I learned today)