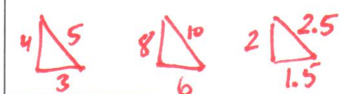
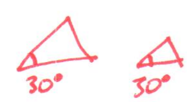
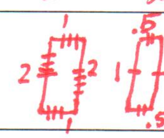
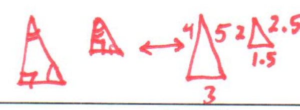
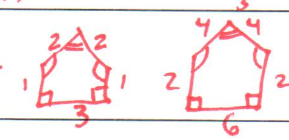
RATES, RATIOS AND PROPORTIONS 10

Name: Key **Date:**________ **Period:**______

SECTION 10.6 SCALING AND SIMILARITY

VOCABULARY

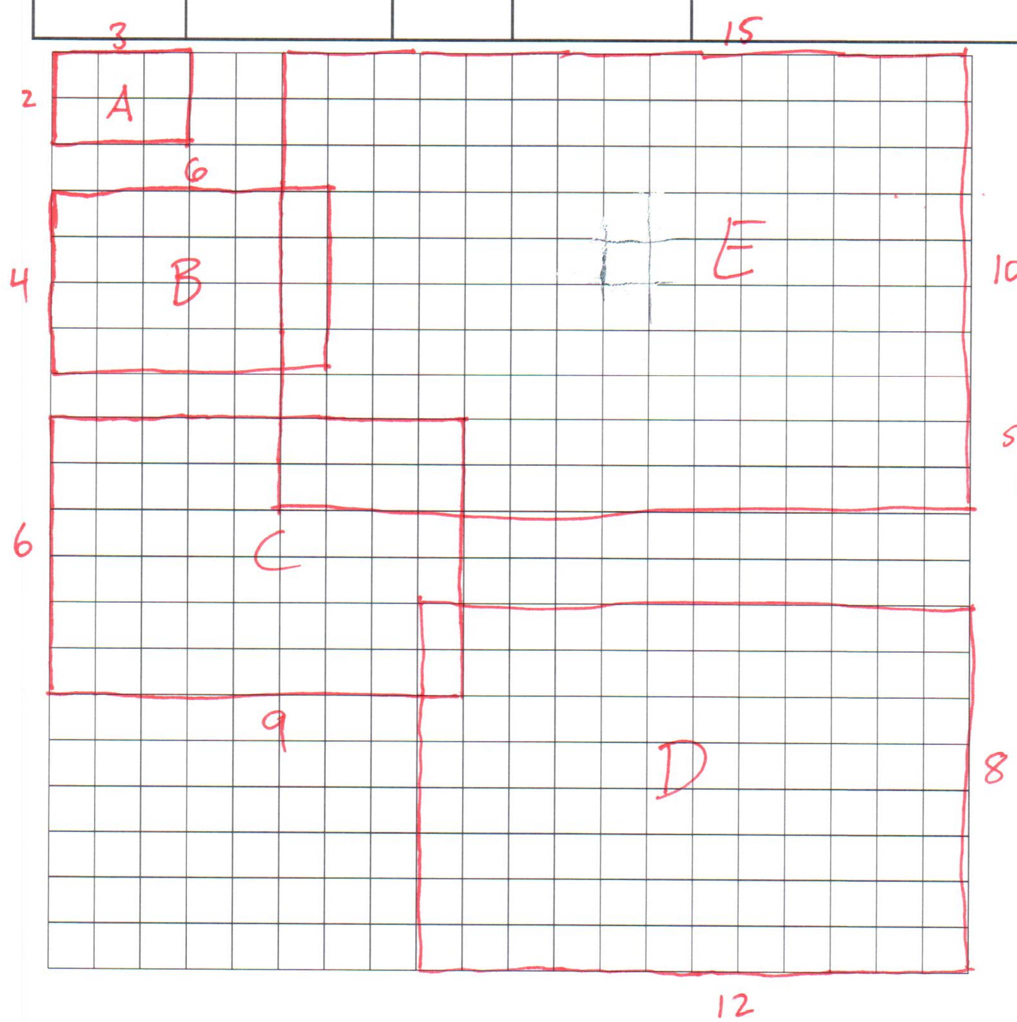
DEFINITION	EXAMPLE
Scaling: the process of reducing or expanding a shape proportionally	
Similar: figures whose corresponding angles are equal in measure and corresponding side lengths have the same ratio.	
Corresponding angles: Angles in the same relative position in polygons - have equal measures in similar polygons.	
Corresponding sides: Sides in the same relative position in polygons - have same ratios in similar polygons	
Triangle Similarity Theorem: If two triangles have corresponding angles with the same measure, the ratios of the corresponding sides are the same (and vice versa)	
Polygon Similarity Theorem: Two polygons are similar when their corresponding angles have the same (equal) measure and their corresponding sides have the same ratio.	

Big Idea: How are scaling and similarity related?

EXPLORATION 1

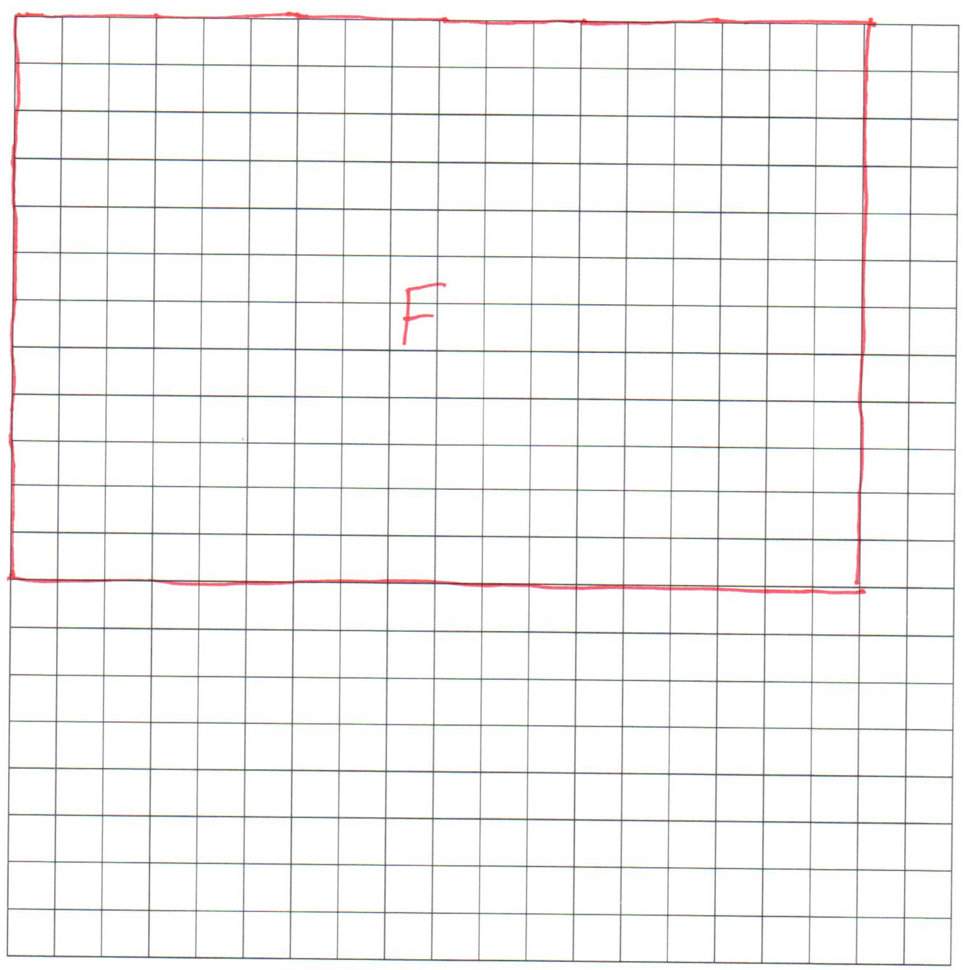
a. Draw rectangle A in the upper left corner of the grid paper. Apply the scale factor to each new rectangle. Draw rectangles A through E and label them appropriately. Draw rectangle E on the grid paper on the next page. Use the new rectangles to complete the tables.

Figure	Dimensions	Scale Factor	Perimeter	Ratio of New Perimeter to Original Perimeter
A	2 x 3	1	10 units	—
B	4 × 6	2	20 units	$\frac{20}{10} = 2$
C	6 × 9	3	30 units	$\frac{30}{10} = 3$
D	8 × 12	4	40 units	$\frac{40}{10} = 4$
E	10 × 15	5	50 units	$\frac{50}{10} = 5$



should be on next page.

Figure	Dimensions	Scale Factor	Area	Ratio of New Area to Original Area
A	2 x 3	1	6 sq. units	—
B	4 × 6	2	24 sq units	$\frac{24}{6} = 4 = 2^2$
C	6 × 9	3	54 sq. units	$\frac{54}{6} = 9 = 3^2$
D	8 × 12	4	96 sq. units	$\frac{96}{6} = 16 = 4^2$
E	10 × 15	5	150 sq. units	$\frac{150}{6} = 25 = 5^2$
F	12 × 18	6		



b. Using your information, make a prediction for the perimeter and area for a new rectangle F with a scale factor of 6.

Perimeter:

$10 \cdot 6 = \boxed{60 \text{ units}}$

Area:

$6 \cdot 6^2 = \boxed{216 \text{ sq. units}}$

c. What is the relationship between the scale factor and the perimeters of the rectangles?

Scale factor of figures $= x$

Scale factor of perimeters $= x$

Linear measurements have the same scale factor.

d. What is the relationship between the scale factor and areas of the scaled rectangles?

Scale factor of dimensions (linear) of figures $= x$

Scale factor of areas $= x^2$

The ratio of change in area is the scale factor squared.

(recall that the units are also squared for area)

Given a rectangle of measure n by m and a new rectangle scaled by factor k, then

e. Write a rule to compare the new perimeter, P_{new}, of the rectangle, with the old perimeter, P_{old}.

$P_{old} = 2n + 2m = 2(n+m)$

$P_{new} = 2(kn) + 2(km) = 2k(n+m) = k \cdot 2(n+m)$

$\boxed{P_{new} = k \cdot P_{old}}$

f. Write a rule to compare the new area, A_{new}, of the rectangle with the old area, A_{old}.

$A_{old} = n \cdot m$

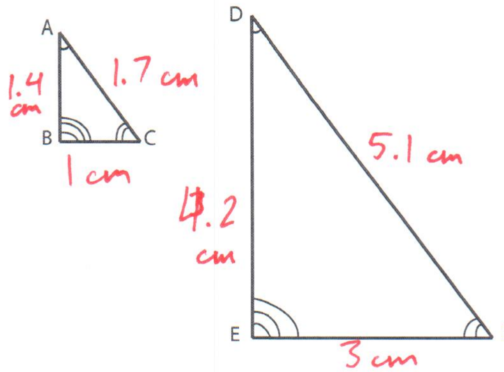
$A_{new} = (nk)(m\cancel{k}k) = n \cdot m \cdot k \cdot k = k^2(n \cdot m)$

$\boxed{A_{new} = k^2 A_{old}}$

EXPLORATION 2

What do you notice about the triangles below?

a. Measure the angles and the side lengths of triangle ABC and triangle DEF.



b. Compare the following angle pairs: A to D, B to E, and C to F. What do you notice?

equal equal equal

c. Compare the side lengths AB to DE, BC to EF, and AC to DF. What do you notice?

ratio of 1 : 3

d. Compute the ratio of each pair of side lengths from part c. What pattern do you notice in these ratios? Do you see a scale factor from triangle *ABC* to triangle *DEF*? Do you see a scale factor from triangle *DEF* to triangle *ABC*? yes, 3

yes, $\frac{1}{3}$

e. Graph the two triangles on a coordinate plane and discuss your findings. How do the area and perimeter compare? Explain using algebraic notation.

Perimeters:

$1.4 + 1 + 1.7 = 4.1$

$4.2 + 3 + 5.1 = 12.3$

Perimeter is 3 times the original.

Area:

$\frac{1.4 \cdot 1}{2} = 0.7$

$\frac{4.2 \cdot 3}{2} = 6.3$

Area is 9 times the original.

f. How do these two triangles relate to the Triangle Similarity Theorem?

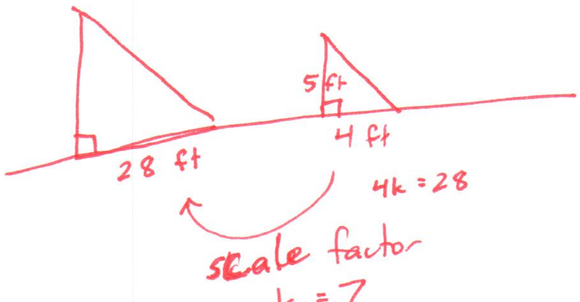
Equal corresponding angles $\leftrightarrow$ same ratio of corresponding sides

g. How do these two triangles relate to the Polygon Similarity Theorem?

Equal corresponding angles and corresponding sides with the same ratios mean the triangles are similar

EXAMPLE 1

In the afternoon, a tree casts a shadow of 28 feet. If a 5 foot post casts a 4 foot shadow next to the tree, what is the height of the tree? Draw a picture to illustrate the problem.



$5 \cdot 7 = \boxed{35 \text{ ft is the tree's height}}$

OR

$$\frac{x}{28} = \frac{5}{4}$$

$$28\left(\frac{x}{28}\right) = \left(\frac{5}{4}\right)28$$

$$x = \frac{140}{4} = \boxed{35 \text{ ft}}$$

PRACTICE EXERCISES

1. Triangle ABC has sides of length 12, 4, and 16.

   a. Triangle DEF has side lengths of 36, 12, and 48. Are these two triangles similar? Explain.

   shortest to longest sides (corresponding sides matched up)

   4, 12, 16

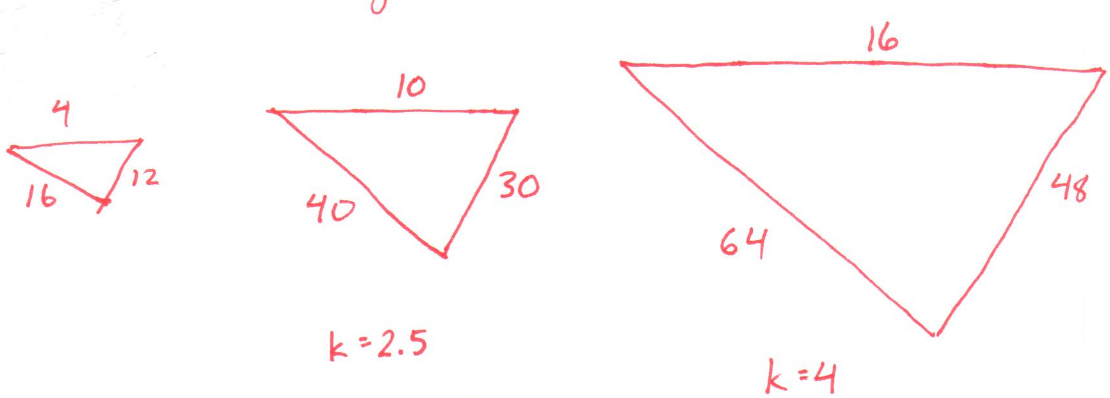
   12, 36, 48

   $\frac{4}{12} = \frac{1}{3}$ $\frac{12}{36} = \frac{1}{3}$ $\frac{16}{48} = \frac{1}{3}$

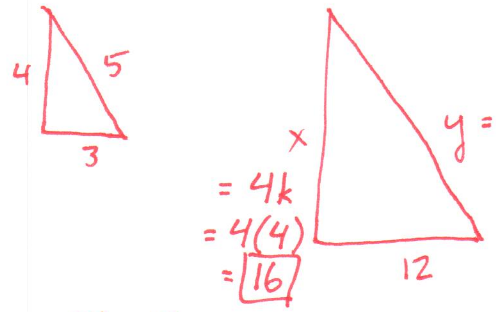
   $\boxed{\text{Yes.}}$ the ratios between corresponding sides are the same.

   b. Draw two triangles similar to triangle ABC, one with scale factor of 2.5 and the other with scale factor of 4.

   (drawings not to scale)



2. Suppose that the sides of ΔABC are 3, 4, and 5 and another triangle, $\Delta A'B'C'$ is similar to ΔABC. The measure of the side of $\Delta A'B'C'$ corresponding to the side of ΔABC with length of 3 has a length of 12. What are the measures of the other two sides of $\Delta A'B'C'$?



$x = 4k = 4(4) = \boxed{16}$

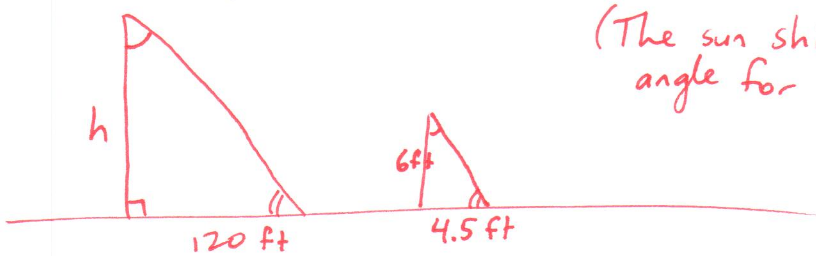
$y = 5k = 5(4) = \boxed{20}$

$3k = 12$

$\frac{3k}{3} = \frac{12}{3}$

$k = 4$

3. In the early afternoon, a building casts a shadow of length 120 ft. At the same time a 6 foot tall person nearby casts a shadow of length 4.5 ft. What is the height of the building? Draw a picture to help illustrate the problem.



(The sun shines at the same angle for both.)

$120\left(\frac{h}{120}\right) = \left(\frac{6}{4.5}\right)120$

$h = \frac{6 \cdot 120}{4.5} = \boxed{160 \text{ ft}}$

OR

$4.5k = 120$

$\frac{4.5k}{4.5} = \frac{120}{4.5}$

$k = \frac{80}{3}$

$h = 6k$

$h = 6\left(\frac{80}{3}\right)$

$\boxed{h = 160 \text{ ft}}$

SUMMARY (What I learned this section)